Calculation of thermal conductivity coefficients of electrons in magnetized dense matter

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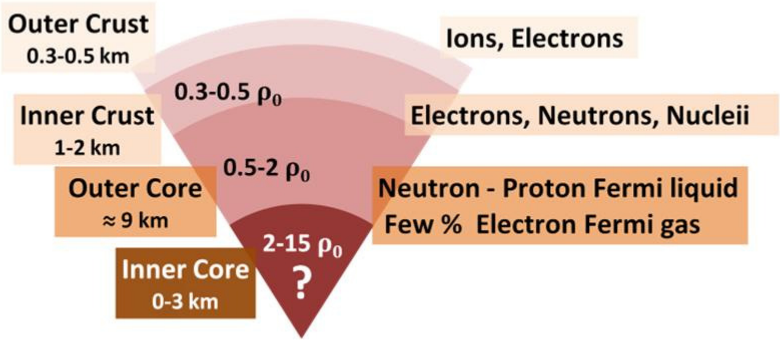
Introduction

Source, RX J	Spin Periods, s	Amplitude/2	Temperature, eV	Absorption line energy, eV
1856.5–3754	7.06	1.5%	60-62	no
0720.4–3125	8.39	11%	85-87	270
1605.3+3249 (RBS 1556)	???	-	93-96	450
1308.6+2127 (RBS 1223)	10.31	18%	102	300
2143.0+0654 (RBS 1774)	9.44	4%	102-104	700
0806.4–4123	11.37	6%	92	460
0420.0–5022	3.45	13%	45	330

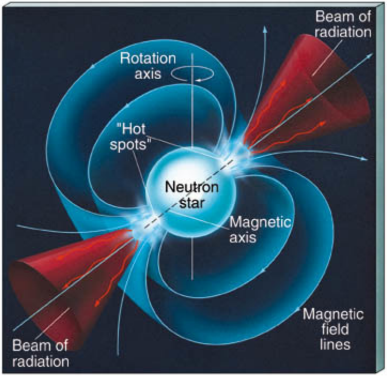
X-ray observations of thermal emission show periodic variabilities in single neutron stars, indicating to the anisotropic temperature distribution. The Magnificent Seven is the informal name of a group of isolated young cooling neutron stars at a distance of 120 to 500 parsecs from Earth. These objects are also known under the names XTINS (X-ray Thermal Neutron Stars).

Structure



Outer Crust
0.3-0.5 km
Ions, Electrons
0.3-0.5 ρ_0
Inner Crust
1-2 km
Electrons, Neutrons, Nucleii
0.5-2 ρ_0
Outer Core
≈ 9 km
Neutron - Proton Fermi liquid
Few % Electron Fermi gas
2-15 ρ_0
?
Inner Core
0-3 km

Influence of the magnetic field



Magnetic field creates anisotropy in heat flux, so heat coefficients are determined by tensor.

Influence of the magnetic field

- $\frac{\sigma \perp}{\sigma \parallel} = \frac{1}{1+(\omega\tau)^2}$. (Flowers, Itoh 1976)
- This dependence on the magnetic field is used in most subsequent papers on this subject $\frac{\lambda_\perp}{\lambda_\parallel} = \frac{1}{1+(\omega\tau)^2}$. (Yakovlev, Urpin 1980)

Influence of the magnetic field

THE MATHEMATICAL THEORY OF NON-UNIFORM GASES

An account of the kinetic theory of viscosity, thermal conduction, and diffusion in gases

by

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Influence of the magnetic field

18.31. *The free-path theory of conduction of heat, and diffusion, in a magnetic field.* We now consider a simple free-path theory of diffusion and heat conduction in an ionized gas at rest in the presence of a magnetic field, using a method somewhat analogous to that of 6.3 and 6.4. As in 18.3, the magnetic field is taken to be parallel to Ox, and in addition it is assumed that the density, temperature, and composition are functions of z alone, and that $X_1 = 0$.

We assume, purely as a convenient rough approximation, that the mean time between successive collisions of a molecule m_s has the same value τ_s, whatever the molecular speed. With this assumption, by an argument similar to that of 5.41, e^{-t/τ_s} is the probability that at any given instant a molecule m_s has travelled without collision for a time at least equal to t.

The number of collisions per unit time experienced by molecules m_s in a volume $\boldsymbol{r}$, $d\boldsymbol{r}$ is $n_s d\boldsymbol{r}/\tau_s$. Let $\chi_s(c_s, z)\, d\boldsymbol{c}_s d\boldsymbol{r}/\tau_s$ denote the number of these which result in a molecule m_s entering the velocity-range $\boldsymbol{c}_s$, $d\boldsymbol{c}_s$; as the notation implies, it is assumed that χ_s depends only on the magnitude of $\boldsymbol{c}_s$, and not on its direction. In a gas in the uniform steady state χ_s is identical with Maxwell's function f_s, since the number of molecules entering any velocity-range through collision is equal to the number leaving. In general χ_s will differ only slightly from f_s.

We consider first diffusion. Since free-path methods seem unable to give an adequate theory of thermal diffusion, arising from inequalities of temperature, we assume that the temperature is uniform.

Influence of the magnetic field

$$\overline{w}_1 = \left(Z_1 - \frac{1}{\rho_1}\frac{\partial p_1}{\partial z}\right)\int_0^\infty \sin\omega_1 t\, e^{-t/\tau_1}\,\frac{dt}{\omega_1\tau_1}$$

$$= \left(Z_1 - \frac{1}{\rho_1}\frac{\partial p_1}{\partial z}\right)\frac{\tau_1}{1+\omega_1^2\tau_1^2}. \qquad \ldots\ldots 3$$

The velocity of diffusion of the molecules m_1 when the magnetic field is absent is found by putting $\omega_1 = 0$ in 3. The presence of the magnetic field accordingly results in a reduction in the velocity of diffusion parallel to Oz in the ratio $1 : (1 + \omega_1^2\tau_1^2)$.

Influence of the magnetic field

Clearly 3 and 4 can only be approximate results; it is actually not possible, with any values of τ_1 and τ_2, for these formulae to be consistent, for all values of H, with the conditions that the gas as a whole should be at rest,* namely

$$n_1 m_1 \bar{v}_1 + n_2 m_2 \bar{v}_2 = 0, \quad n_1 m_1 \overline{w}_1 + n_2 m_2 \overline{w}_2 = 0. \qquad \ldots\ldots 5$$

Consequently results based on 3 and 4 must be treated with reserve; we cannot expect to deduce from them more than the relative order of magnitude of the direct and Hall currents, and the order of magnitude of the reduction in the conductivity. Using 3 and 4, the direct and Hall currents are found to be

$$j_z = \left(Z_1 - \frac{1}{\rho_1}\frac{\partial p_1}{\partial z}\right)\frac{n_1 e_1 \tau_1}{1+\omega_1^2\tau_1^2} + \left(Z_2 - \frac{1}{\rho_2}\frac{\partial p_2}{\partial z}\right)\frac{n_2 e_2 \tau_2}{1+\omega_2^2\tau_2^2} \qquad \ldots\ldots 6$$

$$j_y = \left(Z_1 - \frac{1}{\rho_1}\frac{\partial p_1}{\partial z}\right)\frac{n_1 e_1 \omega_1 \tau_1^2}{1+\omega_1^2\tau_1^2} + \left(Z_2 - \frac{1}{\rho_2}\frac{\partial p_2}{\partial z}\right)\frac{n_2 e_2 \omega_2 \tau_2^2}{1+\omega_2^2\tau_2^2}. \qquad \ldots\ldots 7$$

Boltzmann equation

$$\frac{\partial f}{\partial t} + c_i \frac{\partial f}{\partial r_i} + \frac{e}{m}(E_i + \frac{1}{c}\epsilon_{ikl} c_k B_l)\frac{\partial f}{\partial c_i} + J = 0.$$

For collision integral we take into account only pair collisions:

$$J = J_{ee} + J_{eN} = D \int [f^{'} f_1^{'} (1 - f)(1 - f_1) - f f_1 (1 - f^{'})(1 - f_1^{'})] \times g_{ee} b db d\varepsilon dc_{1i} + \int [f^{'} f_N^{'} (1 - f) - f f_N (1 - f^{'})] \times g_{eN} b db d\varepsilon dc_{Ni}$$

The integration in electron part of the collision integral is performed over the phase space of the incoming particles (dc_{1i}), and the physical space of their arrival ($b\, db\, d\varepsilon$).

Solution method

- Subsequent approximations method:
- Zeroth approximation for electron distribution function: $f_0 = [1 + \exp \frac{m_e v^2 - 2\mu}{2kT}]^{-1}$.
- First approximation for electrons: $f = f_0[1 + \chi(1 - f_0)]$
- $\chi = -A_i \frac{\partial \ln T}{\partial r_i} - n_e D_i d_i \frac{G_{5/2}}{G_{3/2}}$,
- $\chi_N = -A_{Ni} \frac{\partial \ln T}{\partial r_i} - n_N D_{Ni} d_i \frac{G_{5/2}}{G_{3/2}}$
- $G_n(x_0) = \frac{1}{\Gamma(n)} \int_0^\infty \frac{x^{n-1} dx}{1+exp(x-x_0)}$, $x_0 = \frac{\mu}{kT}$ where $G_n(x_0)$ are Fermi integrals.
- Functions A_i, A_{Ni} и D_i, D_{Ni} determine the heat conductivity and diffusion.
- $A_i = A^1 v_i + A^2 \epsilon_{ijk} v_j B_k + A^3 B_i (v_j B_j)$
- $\xi = A^1 + iBA^2$

Solution method

- Sonine polynomial are coefficients of the expansion of the function $(1-s)^{-\frac{3}{2}-1}e^{\frac{xs}{1-s}}$ in powers of s:
  $(1-s)^{-\dfrac{3}{2}-1}e^{\dfrac{xs}{1-s}} = \Sigma S_{3/2}^{(p)}(x)s^p.$
- $S_{3/2}^{(0)}(x) = 1, \quad S_{3/2}^{(1)}(x) = \dfrac{5}{2} - x, \quad S_{3/2}^{(2)}(x) = \frac{35}{8} - \frac{7}{2}x + \frac{1}{2}x^2.$
- For degenerate case we have to seek solution in the form of an expansion in polynomials Q_n, that are orthogonal with weight $f_0(1-f_0)x^{3/2}$ analoguos to Sonine polynomials:
- $Q_0(x) = 1, Q_1(x) = \dfrac{5G_{5/2}}{2G_{3/2}} - x,$
  $Q_2(x) = \frac{35}{8}\frac{G_{7/2}}{G_{3/2}} - \frac{7G_{7/2}}{2G_{5/2}}x + \frac{1}{2}x^2, \quad x = u^2.$

Tensor of heat conductivity in a general form

- $\xi = a_0 Q_0 + a_1 Q_1 + a_2 Q_2$

  $a_0 = a_0^1 + iBb_0^1 \quad a_1 = a_1^1 + iBb_1^1 \quad B^2\, c_0^1 = (a_0^1)_{B=0} - a_0^1$
  $B^2\, c_1^1 = (a_1^1)_{B=0} - a_1^1$

$$q_i = -\lambda_{ik} \frac{\partial T}{\partial r_k},$$

$$\lambda_{ik} = \frac{5}{2} \frac{k^2 T n_e}{m_e} \frac{G_{5/2}}{G_{3/2}} \left\{ \left[a_0^1 - \left(\frac{7}{2} \frac{G_{7/2}}{G_{5/2}} - \frac{5}{2} \frac{G_{5/2}}{G_{3/2}} \right) a_1^1 \right] \delta_{ik} \right.$$

$$- \varepsilon_{ikn} B_n \left[b_0^1 - \left(\frac{7}{2} \frac{G_{7/2}}{G_{5/2}} - \frac{5}{2} \frac{G_{5/2}}{G_{3/2}} \right) b_1^1 \right]$$

$$\left. + B_i B_k \left[c_0^1 - \left(\frac{7}{2} \frac{G_{7/2}}{G_{5/2}} - \frac{5}{2} \frac{G_{5/2}}{G_{3/2}} \right) c_1^1 \right] \right\}$$

Heat conductivity of strongly degenerate electrons in the presence of magnetic field: Lorentz approximation

$$f_0(1-f_0)(u^2 - \frac{5G_{5/2}}{2G_{3/2}}) = -iBf_0(1-f_0)\frac{e\xi}{m_e c}u_i + f_0(1-f_0)n_N\xi\int(1-\cos\theta)g_{eN}bdbd\varepsilon.$$

The function ξ is defined by expression

$$\xi = \frac{u^2 - \frac{5}{2}\frac{G_{\mathbf{5}/\mathbf{2}}}{G_{\mathbf{3}/\mathbf{2}}}}{4\pi n_N\left(\frac{m_e}{2kT}\right)^{3/2}\frac{e^{\mathbf{4}}Z^{\mathbf{2}}}{m_e^{\mathbf{2}}u^{\mathbf{3}}}\Lambda - i\omega} = A^1 + iBA^2.$$

$$\lambda^{(1)} = \frac{2\pi}{3}\frac{m_e^4}{h^3 T}\left(\frac{2kT}{m_e}\right)^{7/2}\left[A^{(1)}(x_0)x_0^{5/2} + \frac{\pi^2}{6}\frac{d^2\left(A^{(1)}x^{5/2}\right)}{dx^2}|_{x=x_\mathbf{0}}\right],$$

$$\lambda^{(2)} = -\frac{2\pi}{3}\frac{m_e^4}{h^3 T}\left(\frac{2kT}{m_e}\right)^{7/2}\left[A^{(2)}(x_0)x_0^{5/2} + \frac{\pi^2}{6}\frac{d^2\left(A^{(2)}x^{5/2}\right)}{dx^2}|_{x=x_\mathbf{0}}\right],$$

The average frequency of electron-ion collisions

The average frequency of electron-ion collisions ν_{ei} is written in the form

$$\nu_{ei} = \frac{4}{3}\sqrt{\frac{2\pi}{m_e}}\frac{Z^2 e^4 n_N \Lambda}{(kT)^{3/2} G_{3/2}}\frac{1}{1+e^{-x_0}}.$$

In the limiting cases it is expressed as

$$\nu_{ei} = \frac{4}{3}\sqrt{\frac{2\pi}{m_e}}\frac{Z^2 \mathrm{e}^4 n_N \Lambda}{(kT)^{3/2}} \qquad (ND)$$

$$= \frac{32\pi^2}{3} m_e \frac{Z^2 \mathrm{e}^4 \Lambda n_N}{h^3 n_\mathrm{e}} \qquad (D).$$

The average time τ_{ei} between (ei) collisions is the inverse value of ν_{ei}, and is written as

$$\tau_{nd} = \frac{1}{\nu_{nd}} = \frac{3}{4}\sqrt{\frac{m_e}{2\pi}}\frac{(kT)^{3/2}}{Z^2 \mathrm{e}^4 n_N \Lambda},$$

$$\tau_d = \frac{1}{\nu_d} = \frac{3h^3 n_e}{32\pi^2 m_e Z^2 e^4 \Lambda n_N}.$$

Heat conductivity of strongly degenerate electrons in the presence of magnetic field: Lorentz approximation

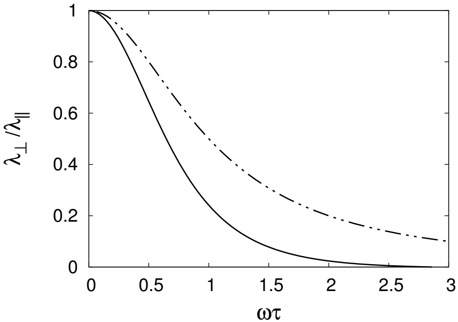
Coefficients of heat conductivity, obtained by solving Boltzmann equation in Lorentz approximation:

$$\lambda^{(1)} = \frac{5\pi^2}{6}\frac{k^2 T n_e}{m_e}\tau_d \left\{ \frac{1}{1+\omega^2\tau_d^2} - \frac{6}{5}\frac{\omega^2\tau_d^2}{(1+\omega^2\tau_d^2)^2} - \frac{\pi^2}{10}\left[\frac{1}{1+\omega^2\tau_d^2\left(\frac{x^3}{x_0^3}\right)}\right]'' \Bigg|_{x=x_0} \right\},$$

$$\lambda^{(2)} = -\frac{4\pi^2}{3}\frac{k^2 T n_e}{m_e}\frac{\tau_d^2\omega}{B} \left\{ \frac{1}{1+\omega^2\tau_d^2} - \frac{3}{4}\frac{\omega^2\tau_d^2}{(1+\omega^2\tau_d^2)^2} - \frac{\pi^2}{16}\left[\frac{1}{1+\omega^2\tau_d^2\left(\frac{x^3}{x_0^3}\right)}\right]'' \Bigg|_{x=x_0} \right\},$$

$$\lambda^{(3)} = \frac{5\pi^2}{6}\frac{k^2 T n_e}{m_e}\tau_d$$

Heat conductivity of strongly degenerate electrons in the presence of magnetic field: Lorentz approximation



The plots of the ratio $\lambda_\perp / \lambda_\parallel$ as a function of $\omega\tau$ are presented for phenomenologically obtained heat conductivity (dash-dot line) for comparison with heat conductivity obtained by the solution of Boltzmann equation in Lorentz appoximation (solid line) with $kT = 0.09E_f$ At $\omega\tau = 1.5$ exact value 4 times smaller than at phenomenological curve.

Conclusion

- We obtain, for the first time, in three polynomial approximation, with account of electron-electron collisions, analytical expressions for the heat conductivity tensor for non-degenerate electrons, in presence of a magnetic field.
- For strongly degenerate electrons we obtain, for the first time, an asymptotically exact analytical solution for the heat conductivity tensor in presence of a magnetic field. This solution has considerably more complicated dependence on the magnetic field than those in previous publications, and gives several times smaller relative value of a thermal conductivity across the magnetic field at $\omega\tau \gtrsim 0.8$.

Thank you for your attention!