The Jet Magnetic Flux

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What we need to produce jets?

- The ordered magnetic field
- The rotating black hole
- The accreting material

Blandford-Znajek process $\rightarrow$ BH rotational energy extraction
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\[ P_{tot} = \frac{\Omega^2}{\pi^2 c} \Psi_{tot}^2 \]
(Beskin 2010)

\[ \Psi_{tot} \propto 50(\dot{M} r_g^2 c)^{1/2} \]
(Zamaninasab+ 2014)

Blandford-Znajek process \rightarrow BH rotational energy extraction
What we need to produce jets?

- The ordered magnetic field
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Need estimates for:
- Magnetic field $B$
- Particle number density $n$

Blandford-Znajek process $\rightarrow$ BH rotational energy extraction
Core-shift measurement

Equipartition assumption

Blandford-Konigl scalings

(Onanov 1998, see also Hirotani 2005, O’Sullivan & Gabuzda 2009, Nokhrina+ 2015)
Which physical parameters we can imply basing on the observations?

Core-shift effect:

Can be measured, for instance, in mas GHz

\[
v_1 > v_2, \quad v_2, \quad \tau = 1
\]
Which physical parameters we can imply basing on the observations?

Equipartition:

\[ \sigma = \frac{B^2}{4\pi nmc^2\Gamma^2} \]

\[ dn = k_e \gamma^{-p} d\gamma \]

\[ \Sigma = \frac{\Gamma B^2 f(2)}{4\pi n_{rad} mc^2 \ln(\gamma_{\text{max}}/\gamma_{\text{min}})} \]
Which physical parameters we can imply basing on the observations?

Basing on the observations,

\[
\sigma = B_2 \frac{4\pi n m c^2 \Gamma_2 d}{\Sigma} = k \varepsilon \gamma - p d \gamma \Sigma = B_2 f(2) 4\pi nr^2 c^2 ln \frac{\Gamma}{\gamma_{max}} \gamma_{min}
\]

Beskin & Nokhrina 2006

Tchekhovskoy, McKinney & Narayan 2009
Which physical parameters we can imply basing on the observations?

Blandford-Konigl (1979) model $B \propto r^{-1}$ and $n \propto r^{-2}$

+ Gould (1979) model for the spherical self-absorbed sources
Which physical parameters we can imply basing on the observations?

Blandford-Konigl model + synchrotron self-absorbed source model provides

\[ \nu_{\text{obs}} \propto r^{-1} \]

Sokolovsky+ 2011 supports it.
Core-shift measurement

Equipartition assumption

Blandford-Konigl scalings

\[ B \sim 1G \]

\[ n \sim 10^3 \text{ cm}^{-3} \]

(Lobanov 1998, see also Hirotani 2005, O’Sullivan & Gabuzda 2009, Nokhrina+ 2015)
Why non-equipartition is probably not valid?

- **Kellermann & Pauliny-Toth 1969**: the idea of the inverse Compton catastrophe and the limiting intrinsic brightness temperature
  \[ T_{br} \approx 10^{12}\text{K} \]
- **Readhead 1994**: the equipartition brightness temperature
  \[ T_{br} \approx 10^{11.5}\text{K} \]
- **However**: recent observations of radio cores by Gomez+ 2016, Kovalev+ 2016, Lisakov+ 2017 provide
  \[ T_{br} > 7 \times 10^{12}\text{K} \]
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Core-shift measurement

 Equipartition assumption

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Core-shift measurement

Equi-partition assumption

Blandford-Konigl scalings

Core-shift measurement

Flux (Tb) measurement

Blandford-Konigl scalings
Can we estimate independently the B and n?

Zdziarski, Sikora, Pjanka & Tchekhovskoy, 2015: let us use the flux measurement + core-shift measurement => independent evaluation of B and n in the radio core region. The result is that the magnetic field is nearly equipartition. However, the flux measurements correspond to the sub-equipartition limit.
1. Core-shift effect;  
2. Brightness temperature measurement;  

\[
\left( \frac{B_{uni}}{G} \right) = 7.4 \times 10^{-4} \frac{\Gamma \delta}{1 + z} \left( \frac{\nu_{obs}}{GHz} \right) \left( \frac{T_{b,obs}}{10^{12} K} \right)^{-2}
\]

\[
\left( \frac{n}{cm^{-3}} \right) = 8.2 \times 10^3 \frac{\Gamma \sin^2 \varphi (1 + z)^7}{2\chi \delta^4} f(2) \times
\]

\[
\times \left( \frac{D_L}{Gpc} \right)^{-1} \left( \frac{\Phi}{mas \ GHz} \right)^{-1} \left( \frac{\nu_{obs}}{GHz} \right)^2 \left( \frac{T_{b,obs}}{10^{12} K} \right)^4
\]
Magnetization of the radiating region: the ratio of magnetic energy flux to the plasma particle energy flux

\[ \Sigma = 7.7 \times 10^{-5} \frac{2\chi \Gamma^2 \delta^6}{\sin^2 \varphi (1 + z)^9} \frac{F(2)}{f(2)} \times \]

\[ \times \left( \frac{D_L}{G\text{pc}} \right) \left( \frac{\Phi}{\text{mas GHz}} \right) \left( \frac{T_{b,\text{obs}}}{10^{12} K} \right)^{-8} \]

These are the upper limits for B and \( \Sigma \), and the lower limit for n.
BL Lac and 3C273

- BL Lac (Gomez+ 2016)
  - $T_{b,obs} = 7.9 \times 10^{12}$K at $\nu_{obs} = 15$ GHz
  - $B_{uni} = 3.3 \times 10^{-2}$G

- 3C273 (Kovalev+ 2016)
  - $T_{b,obs} = 13 \times 10^{12}$K at $\nu_{obs} = 4.8$ GHz
  - $B_{uni}(high) = 8.1 \times 10^{-3}$G
  - $B_{uni}(low) = 0.13$ G
    - (for $T_b=4 \times 10^{12}$K at $\nu_{obs} = 16.7$ GHz)
What about the total magnetic flux in a jet?

• Dynamically important magnetic field – regulate the accretion rate
What about the total magnetic flux in a jet?


• Dynamically important magnetic field – regulate the accretion rate

• $\Psi_{MAD} \sim 50 \sqrt{\dot{M} r_g^2 c}$
What about magnetic flux in a jet?

Zamaninasab+ 2014:

\[
\frac{B_P}{B_\varphi} \propto a \frac{R_j}{r_g}
\]

From CS+BK+E the measured field \( B_\varphi \), and the flux

\[
\Psi \propto R_j^2 B_P \propto MR_j B_\varphi
\]

Let us account for the transversal jet structure.
Non-uniform model

• Can be obtained solving the non-linear Grad-Shafranov equation on the flux function $\Psi$. It can be done analytically under certain assumptions: self-similarity, or force-free flow (plasma inertia = 0), or effectively 1D – the cylindrical magnetic surfaces configuration.

• The latter is a good approximation for the well-collimated jets, or a slice of a jet where we may neglect by the opening angle on the interesting for us scales.
Non-uniform model: some analytical results

\[ B_P = \frac{\nabla \Psi \times e_\varphi}{2\pi r} \]

\[ B_\varphi = -\frac{2I}{r} e_\varphi \]

\[ E = -\frac{\Omega_F}{2\pi} \nabla \Psi \]
Non-uniform model: some analytical results

\[ B_P = \frac{\nabla \Psi \times e_\phi}{2\pi r} \]

\[ B_\phi = -\frac{2I}{r} e_\phi \]

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Useful relations:

\[ E = B_P \Omega_F r \]
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Useful relations:  
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From the condition of flux freezing one may obtain (Lyubarsky 2009):  
\[ B_\varphi \approx B_P \Omega_F r \]
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\[ B_\phi^2 - E^2 \approx \frac{B_\phi^2}{\Gamma^2} \]
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For the constant current density \( j \)

\[ I = \int j r dr \propto r^2 \]

\[ B_\phi \propto r \]

For the zero current density

\[ B_\phi \propto r^{-1} \]
Non-uniform model: numerical results

The solution may be obtained doing the numerical simulations:

Tchekhovskoy & Bromberg 2016
Non-uniform model: numerical results

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The central core with constant poloidal magnetic field $B_P$ and linearly growing toroidal magnetic field $B_\phi$.

Tchekhovskoy & Bromberg 2016
Non-uniform model: numerical results

The solution may be obtained doing the numerical simulations:

The outer flow with the poloidal magnetic field $B_p \propto r^{-2}$ and the toroidal magnetic field $B_\phi \propto r^{-1}$. 

Tchekhovskoy & Bromberg 2016
Non-uniform model: numerical results

The solution may be obtained doing the numerical simulations:

The size of a central core

$R_0 \approx R_L$

At the central core boundary

$B_P = B_\varphi = B_0$

and we call it the magnetic field amplitude.

Tchekhovskoy & Bromberg 2016
Non-uniform model: analytical results
Non-uniform model: analytical results

The central core:

\[ n \approx \text{const} \]
\[ B_p \approx \text{const} \]
\[ B_\varphi \propto r \]
\[ \Gamma \approx \text{const} \]
Non-uniform model: analytical results

The outer flow:

\[ n \propto r^{-2} \]
\[ B_P \propto r^{-2} \]
\[ B_{\varphi} \propto r^{-1} \]
\[ \Gamma \propto r \]

Nokhrina+ 2015
Non-uniform model: analytical results

The central core size $R_0 \approx 5R_L$

The magnetic field amplitude $B_0$
Non-uniform model

• The non-uniform n and B distribution leads to non-uniform synchrotron emission
  
  \[ \rho = 4\pi(1.5)^{p-1/2} a(p) a k' e \left( \frac{v'_B}{v'} \right)^{(p+1)/2} \]

  and effective absorption
  
  \[ \kappa = c(p) r_0^2 k' e \left( \frac{v_0}{v'} \right) \left( \frac{v'_B}{v'} \right)^{(p+1)/2} \]

  coefficients (important).

• Different boosting Lorentz factors across the jet cross-section (not important, Nokhrina 2017).
Non-uniform model – B-field

For jets with small viewing angles calculation of the observed flux

\[ S_{\nu} = \frac{\delta^3}{d^2} \int \Omega' \frac{\hbar \nu' \rho' dV'}{e^{-\int \kappa' ds'}} \]

can be done analytically. We use the measurements of the brightness temperature for BL Lac (Gomez+ 2016) and 3C273 (Kovalev+ 2016).

BL Lac → \( \varphi = 0.1 \)

3C273 → \( \varphi = 0.067 \)

(using measurements of \( \beta_{app} \) by Lister+ 2013, and Doppler factor by Jorstad+ 2005 and Cohen+ 2007).
Finally, we obtain the following expression for the magnetic field amplitude

\[
\left( \frac{B_0}{G} \right) = 6.4 \times 10^{-4} \Gamma \left( \frac{R_{jet}}{R_L} \right) \frac{\delta}{1 + z} \left( \frac{\nu_{obs}}{\text{GHz}} \right) \left( \frac{T_{b,obs}}{10^{12} K} \right)^{-2}
\]

Compare with the uniform source

\[
\left( \frac{B_{uni}}{G} \right) = 7.4 \times 10^{-4} \Gamma \frac{\delta}{1 + z} \left( \frac{\nu_{obs}}{\text{GHz}} \right) \left( \frac{T_{b,obs}}{10^{12} K} \right)^{-2}
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\]
The expression for the amplitude of non-uniform magnetic field depends on unknown radius of the light cylinder, which is defined by the field lines rotation rate

\[ R_L = \frac{c}{\Omega_F} \]

Thus, the amplitude magnetic field is not known directly from the observations (unlike uniform non-equipartition magnetic field amplitude).

However, one may calculate the flux in a jet and compare it with the MAD flux, thus estimating the lower limit for \( \Omega_F \) and rotational rate

\[ a = \frac{r_g}{R_L} \]
The non-uniform jet model provides readily the expression for the flux

\[ \Psi = 2.7 B_{uni} R_j \frac{r_g}{a} \left[ 1 + 2 \ln \frac{R_j}{ar_g} \right] \]

Here we used the proportionality of amplitude field \( B_0 \) (can not be estimated independently of \( a \)) and uniform field \( B_{uni} \) (can be estimated independently of \( a \)).

The weak dependence of the expression in square brackets of \( a \) allows to use it to estimate \( a \) comparing the observed flux and MAD flux.
• Magnetic flux predicted by MAD seems to be the flux upper limit
• MAD flux:

\[ \Psi_{MAD} \approx 50 \sqrt[\eta_c]{\frac{L_{acc}r_g^2}{\eta_c}} \]

\[ a \geq \frac{2.7B_{uni}R_jr_g}{\Psi_{MAD}} \]

• However \( R_j \) may be underestimated through observed angular size

\[ R_j = \frac{\theta_{obs}D_L}{(1 + z)^2} \]
BL Lac

\[ M = 1.7 \times 10^8 M_\odot \text{ (Woo & Urry 2002)} \]

\[ L_{\text{acc}} = 1.5 \times 10^{45} \text{ erg s}^{-1} \text{ (Zamaninasab+ 2014)} \]

\[ \Psi_{MAD} = 9.2 \times 10^{32} \text{ G cm}^2 \]

\[ B_{\text{uni}} = 3.3 \times 10^{-2} \text{ G} \text{ (Nokhrina 2017)} \]

\[ \theta_{\text{obs}} \geq 21 \text{ mas} \text{ (Gomez+ 2016)} \]

\[ a = 0.5 \]
3C 273

$M = 10^9 M_\odot$ (Woo & Urry 2002)

$L_{acc} = 1.38 \times 10^{48}$ erg s$^{-1}$ (Punsley & Zhang 2011, Torrealba+ 2012)

$\Psi_{MAD} = 1.6 \times 10^{35} G \text{ cm}^2$

$B_{uni} = 0.13 G$

$\theta_{obs} \geq 275 \text{ mas}$ (Kovalev+ 2016)

$a = 0.01$
Conclusions

• Using the extreme brightness temperatures we obtain the non-equipartition magnetic field for the uniform model

\[ B_{uni} \approx 10^{-2} \, G \]

• The non-uniform transversal jet structure provides the estimate for the magnetic flux through observable values and effective rotational rate

\[ a = \frac{r_g}{R_L} \]

• Comparison of the flux depending on \( a \) and the flux predicted by MAD may give a clue on how fast the black hole rotates.
Thank you!