Influence of a plasma on the black hole shadow

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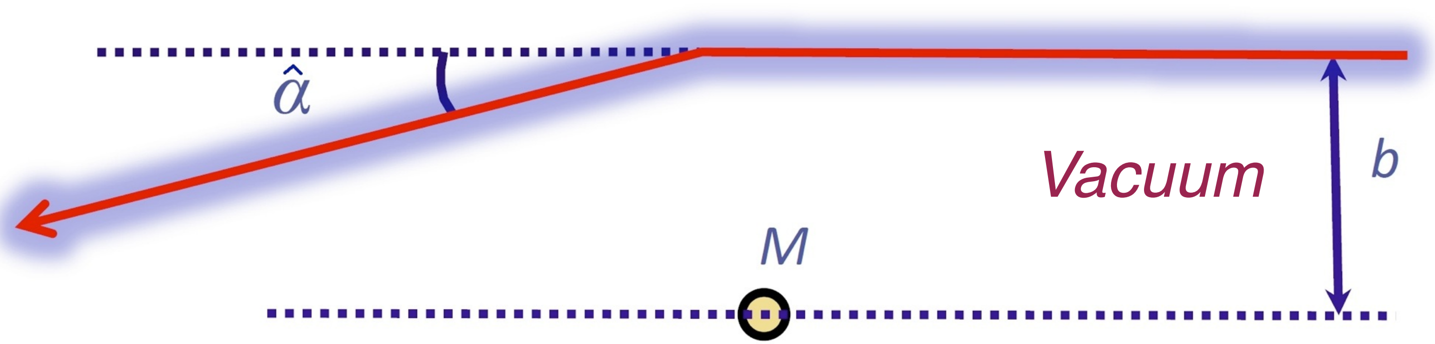
Vacuum:

Einstein angle:

$$\hat{\alpha} = \frac{4GM}{c^2 b} = \frac{2R_S}{b}$$

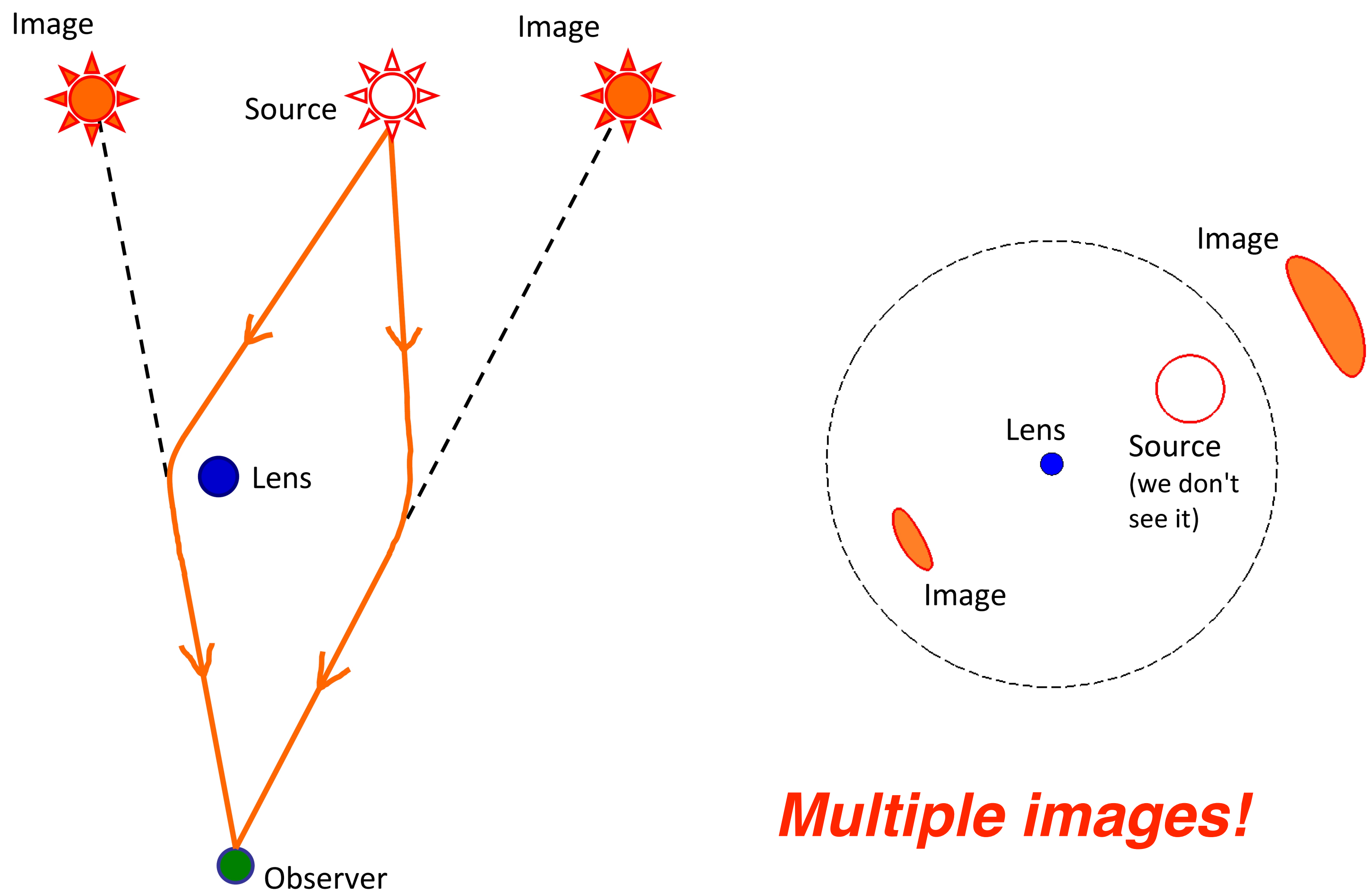
$$b \gg R_S = \frac{2GM}{c^2}$$



Deflection angle of the photon in vacuum does not depend on the photon frequency (or energy). Deflection in vacuum is **achromatic**.

Gravitational lensing in vacuum is achromatic.

Gravitational lensing in vacuum, the simplest scheme



Multiple images!

At this picture there is the example of the simplest model of **Schwarzschild point-mass lens**. Observer sees **two** point **images** of source **instead of one** single real point source.

Plasma:

How is this situation changed in presence of plasma?

In outer space, **the rays of light travel through the plasma**.

In plasma, photons undergo various effects.

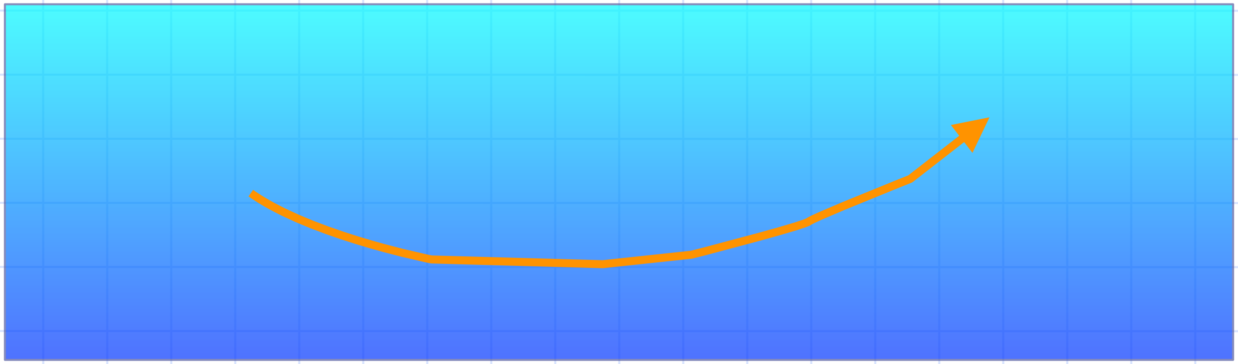
For gravitational lensing the main interest is the **change in the angle of deflection** of the light ray.

Due to dispersion properties of plasma, we may expect that **chromatic effects will arise**.

Refraction

Light rays in a transparent, inhomogeneous medium propagate along curved trajectories. This phenomenon is called refraction and is well known from everyday life.

The bending of the light rays due to refraction is not related to relativity or gravity and occurs only if the medium is optically non-homogeneous.



$n=n(x)$

'Non-homogeneous' means here that the refractive index depends explicitly on space coordinates

Simplified consideration of the problem: ’linearized' combination of gravity and plasma

The simplest way to add plasma to the problem of gravitational lensing is to assume **both deflection angles** due to gravity and due to refraction in the plasma **small**.

Total deflection is:

vacuum gravitational deflection

with using of the linearized theory of gravitation (approximate Einstein formula)

+

refractive deflection in non-homogeneous plasma

(assuming that the refractive index differs only slightly from unity (vacuum))

In this approximation, the joint action of gravitation and refraction was considered from the 1960s, with reference to the propagation of radio signals in the solar corona.

Why we are interested in more rigorous approach

1) **Black holes** and other compact objects **surrounded by a plasma**.

Deflection angles are not small, the linearized theory of gravity is insufficient.

2) The interesting question is whether the **gravitational deflection itself** changes in the presence of a **medium** around a gravitating body instead of a vacuum.

Also, there are many more complex lens models (not just a point mass).

Geometrical optics in presence both gravity and plasma

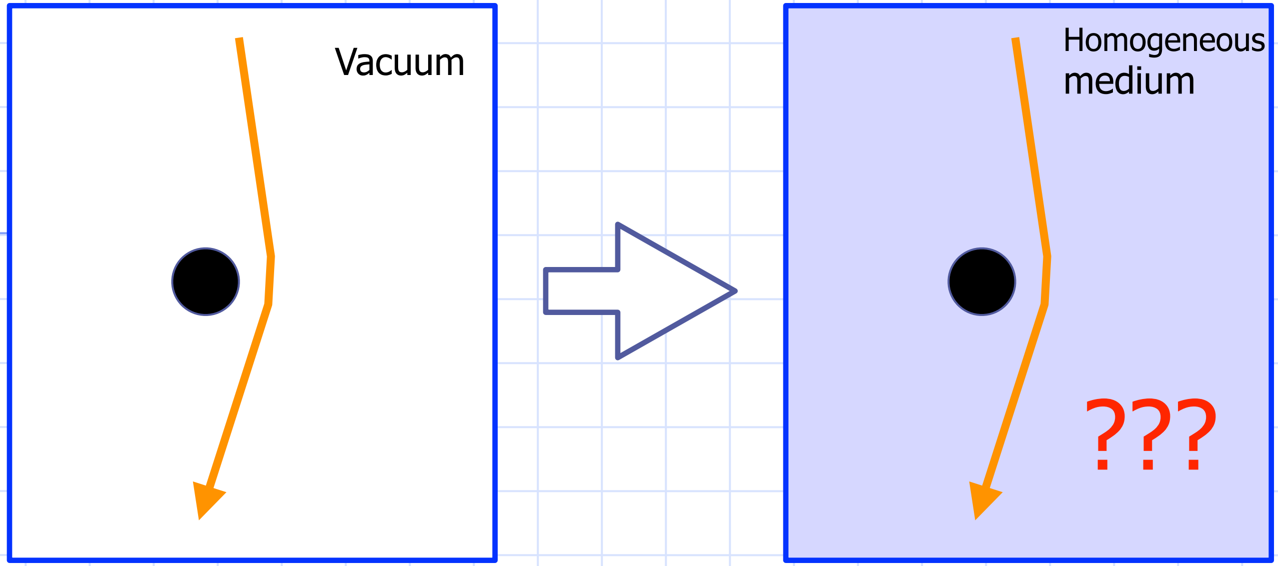
We need general theory for geometrical optics in arbitrary medium (dispersive or not) in curved space-time (in presence of gravity).

J.L. Synge, Relativity: The General Theory (1960)
The first self-consistent approach for geometrical optics in medium in presence of gravity. See also:
J. Bicˇa´k and P. Hadrava (1975)

The first monograph completely devoted to review of general-relativistic ray optics in medium:
V. Perlick, Ray Optics, Fermat's Principle, and Applications to General Relativity (2000)

Propagation of electromagnetic waves in magnetised plasma:
R.A. Breuer and J. Ehlers (1980a, 1980b, 1982)
A. Broderick and R. Blandford (2003, 2004)

On language of gravitational lensing the problem of plasma influence is considered for the first time:
P.V. Bliokh and A.A. Minakov, Gravitational Lenses [Russian] (1989).



is the gravitational deflection of the light rays in the medium the same as in vacuum?

We mean here the *homogeneous* medium, so there is no refraction.

Answer is:
deflection is **the same** as in vacuum, if medium is **non-dispersive** (n=const)
And it **differs**, if medium is **dispersive**! (n=n(omega))

The physical reason is a dependence of the wave frequency on space coordinates in presence of gravity (gravitational redshift)

Bisnovatyi-Kogan and Tsupko (2009, 2010)

Gravitational deflection of light rays in presence of homogenous plasma

Refraction index of plasma:

$$n^2 = 1 - \frac{\omega_e^2}{[\omega(r)]^2}, \qquad \omega_e^2 = \frac{4\pi e^2 N_0}{m} = \text{const}$$

Here, the frequency of the photon $\omega(r)$ depends on the spatial coordinate r due to the presence of a gravitational field (the gravitational redshift). We will use the following notation: $\omega(\infty) \equiv \omega$, e is the charge of the electron, m is the electron mass, ω_e is the electron plasma frequency, and $N_0 = \text{const}$ is the electron concentration in a homogeneous plasma. This for-

We have shown for the first time, that the gravitational deflection in homogeneous plasma differs from the vacuum deflection angle, and depends on frequency of the photon:

$$\hat{\alpha} = \frac{2R_S}{b} = \frac{4GM}{c^2 b}$$

in vacuum

$$\hat{\alpha} = \frac{R_S}{b}\left(1 + \frac{1}{1 - (\omega_e^2/\omega^2)}\right)$$

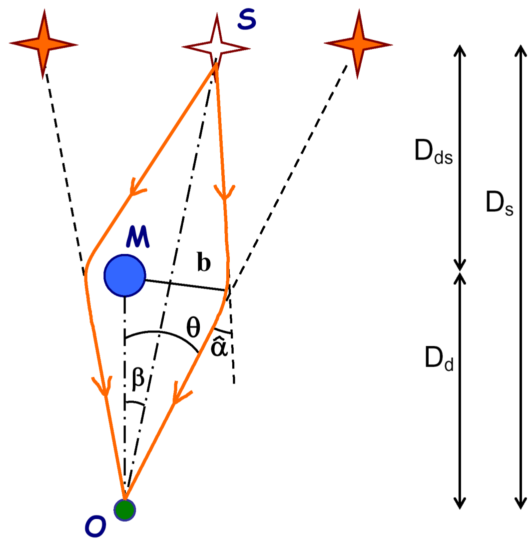
in homogeneous plasma

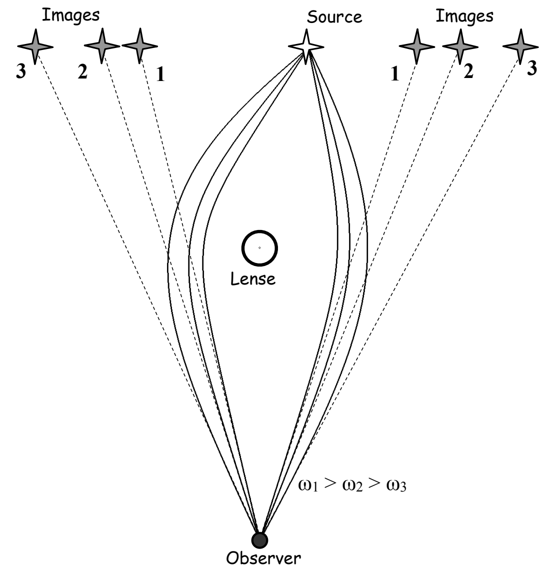
Chromatic gravitational deflection!

Bisnovatyi-Kogan and Tsupko 2009, 2010, 2013

Effect of 'Gravitational radiospectrometer'

Instead of two concentrated images with complicated spectra, we will have two 'rainbow' images, formed by the photons with different frequencies, which are deflected by different angles.

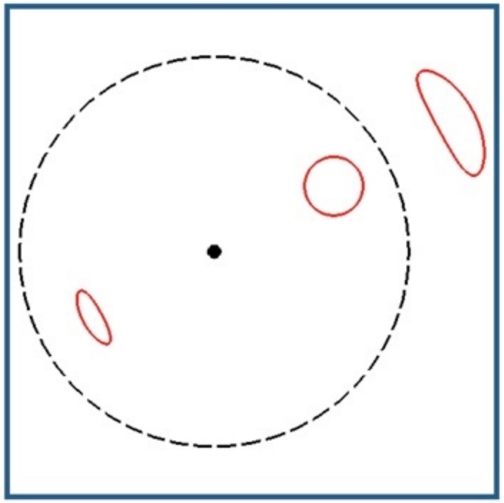




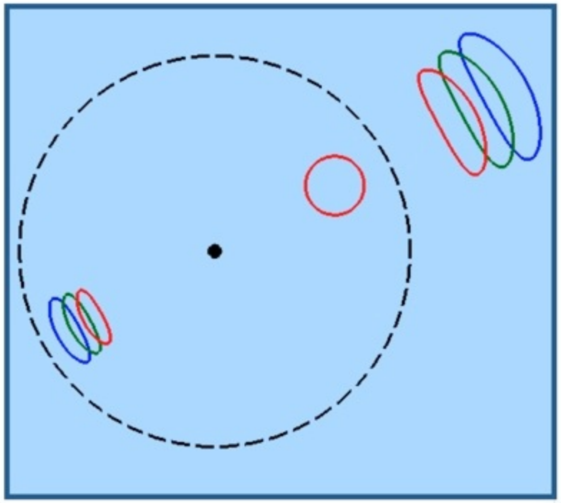
Point-mass gravitational lens in homogeneous plasma: it acts like spectrometer!

Effect is significant only for radiowaves

vacuum



plasma



Different colors mean different wavelengths

Gravitational lensing in non-homogeneous plasma:

In non-homogeneous plasma two effects should be taken into account:

1) Difference of gravitational deflection from vacuum case due to plasma presence (which we have just discussed)

2) Refraction (usually bigger)

Both effects are chromatic in plasma

Total deflection angle (in weak deflection approximation)

$$\hat{\alpha} = \underbrace{\alpha_{einst} + \alpha_{add}}_{\text{gravitational deflection in plasma}} + \alpha_{refr}$$

$$\alpha_{einst} = \frac{2R_S}{b} \qquad \alpha_{add} \propto \frac{R_S}{b}\frac{\omega_e^2}{\omega^2} \qquad \alpha_{refr} \propto \nabla \frac{\omega_e^2}{\omega^2}$$

Vacuum gravitational deflection (Einstein)

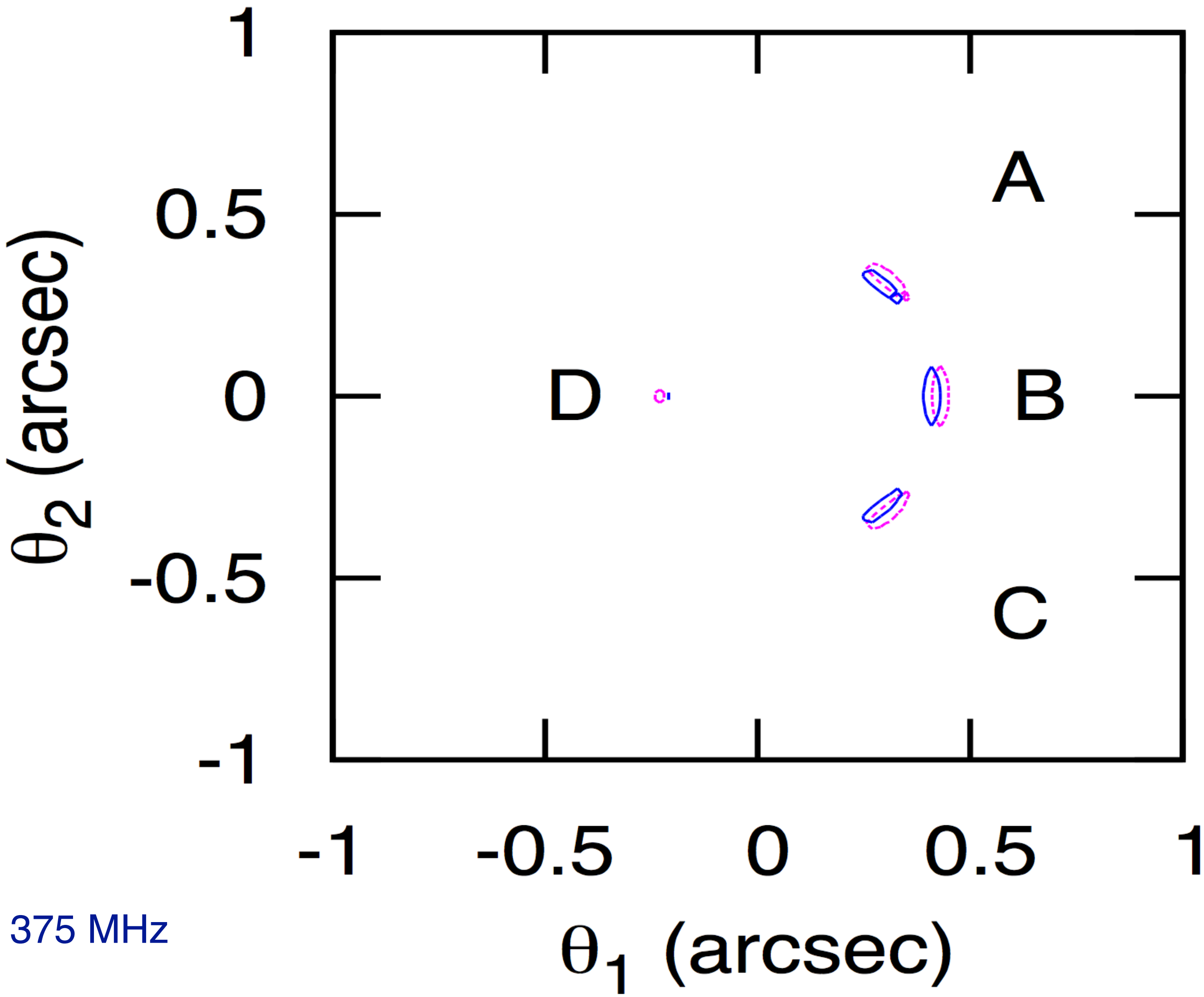
Additional correction to the gravitational deflection due to plasma presence. It depends on the photon frequency. It takes place both in homogeneous and inhomogeneous plasma

The refraction connected with the plasma inhomogeneity. It depends on the photon frequency because the plasma is dispersive medium. This angle equals to zero if the plasma is homogeneous.

Bisnovatyi-Kogan and Tsupko (2009, 2010)

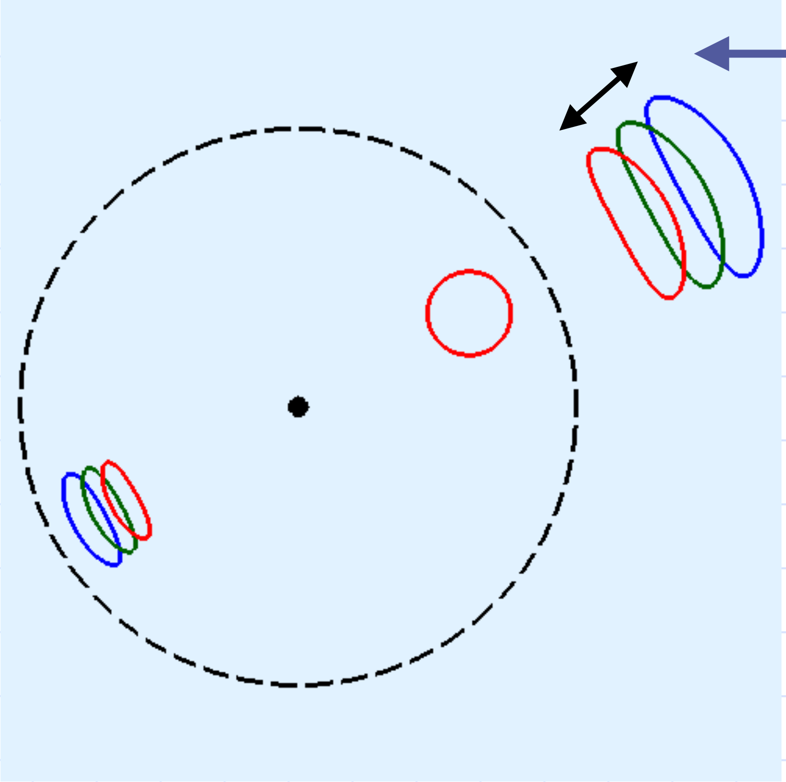
Observational predictions:

- Xinzhong Er, Shude Mao, 2014: angular difference between optical and radio images is up to 10^(-2) arcsec due to presence of inhomogeneous plasma



for 375 MHz

What we propose for observations:



1) Compare observations of strong lens system with multiple images in optical and radio band, or compare observations in two radio bands

2) Shift of angular position of every image can be observed

3) As a result: investigation of plasma properties in vicinity of lens

Shadow of supermassive black holes

SMBH in the center of galaxies, for example, in the center of our galaxy

A distant observer should “see” this black hole as a dark disk in the sky which is known as the “shadow”

For the black hole at the center of our galaxy, size of the shadow is about 53 μas (size of grapefruit on the Moon).

At present, two projects are under way to observe this shadow which would give important information on the compact object at the center of our galaxy. These projects, which are going to use (sub)millimeter VLBI observations with radio telescopes distributed over the Earth, are the Event Horizon Telescope (http://eventhorizontelescope.org) and the BlackHoleCam (http://blackholecam.org).

What is a shadow of a black hole?

We consider a black hole as a region of very strong curved space-time, and the boundary of this region is the event horizon

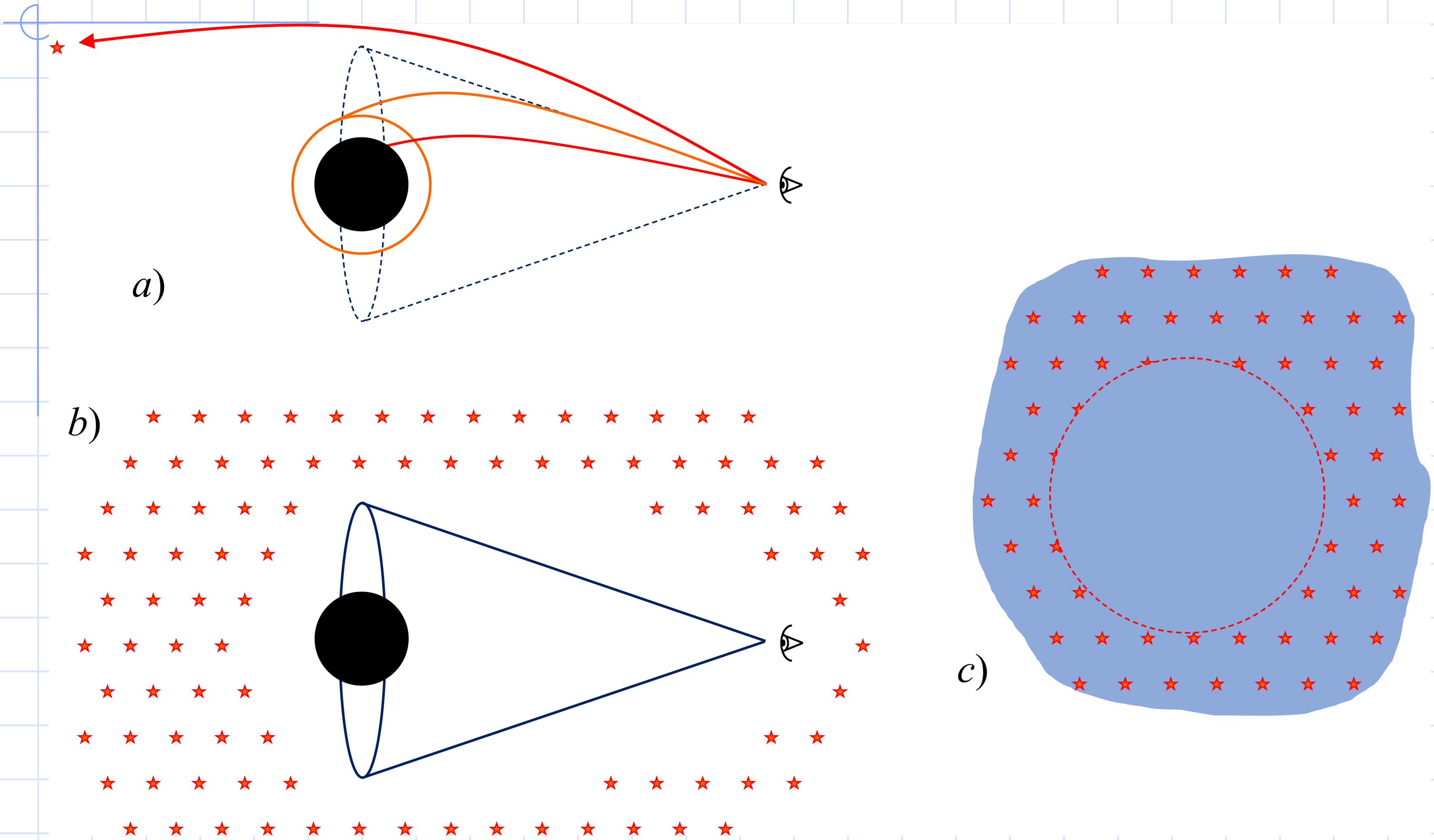
Nothing surprising that we expect to see some black ‘spot’ in the sky there where we suggest a black hole is situated

But why people say that ‘we should see a shadow’ instead of ‘we should see a black hole’?

Answer is that due to STRONG bending of light rays coming to us, we see something very different from ‘real view’ of black hole.

In case of spherically symmetric black hole a difference is only in angular size, in Kerr metric (which is axially symmetric) picture become non-symmetrical

On the theoretical side, the shadow is defined as the region of the observer's sky that is left dark if there are light sources distributed everywhere but not between the observer and the black hole.



Angular size of the shadow of Schwarzschild BH in vacuum

J.L. Synge, MNRAS 131, 463 (1966)

$$\sin^2\alpha_{\rm sh} = \frac{27M^2(1-2M/r_{\rm O})}{r_{\rm O}^2}$$

Schwarzschild radius

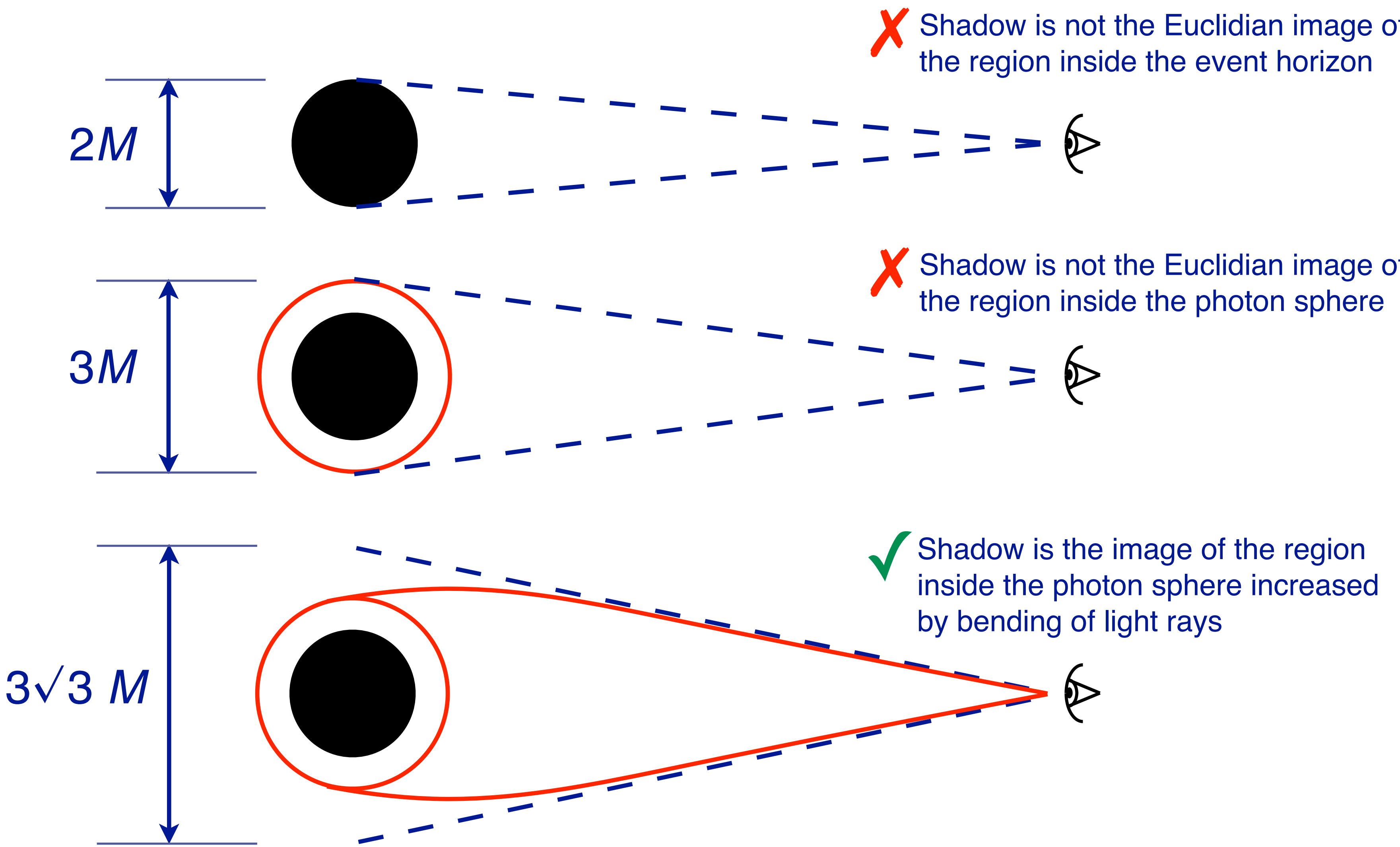
Radius of the photon sphere

$$R_S = 2M, \quad r_{ph} = 3M$$

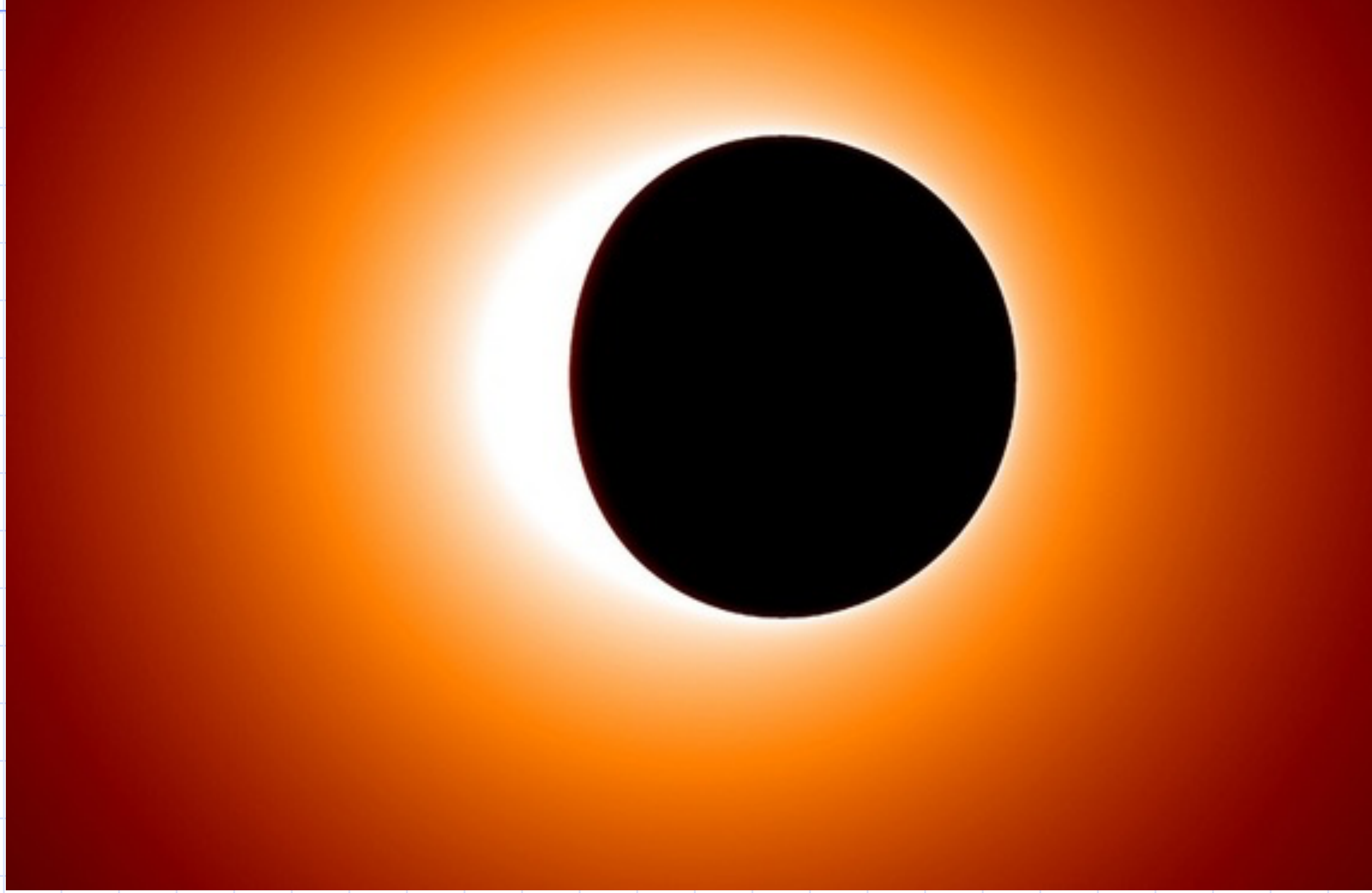
For distant observer, $r_O >> M$:

$$\alpha_{\rm sh} \simeq 3\sqrt{3}\,\frac{M}{r_{\rm O}}$$

Shadow of Schwarzschild black hole



For **rotating** black hole the shadow is not symmetrical although metric is axially symmetric. Shadow is **deformed** and **oblate**.



http://www.space.com/images/i/000/035/394/i02/event-horizon.jpg?1387319865

Analytical calculation of the shadow size and shape in vacuum, other metrics:

Kerr:

J.M. Bardeen, in Black Holes, edited by C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1973), p. 215.

(for observer far away from a black hole)

Class of Plebański-Demiański spacetimes:

A.Grenzebach, V. Perlick, and C. Lämmerzahl, Phys. Rev. D 89, 124004 (2014).
A.Grenzebach, V. Perlick, and C. Lämmerzahl, Int. J. Mod. Phys. D 24, 1542024 (2015).

(including Kerr space-time and arbitrary position of observer)

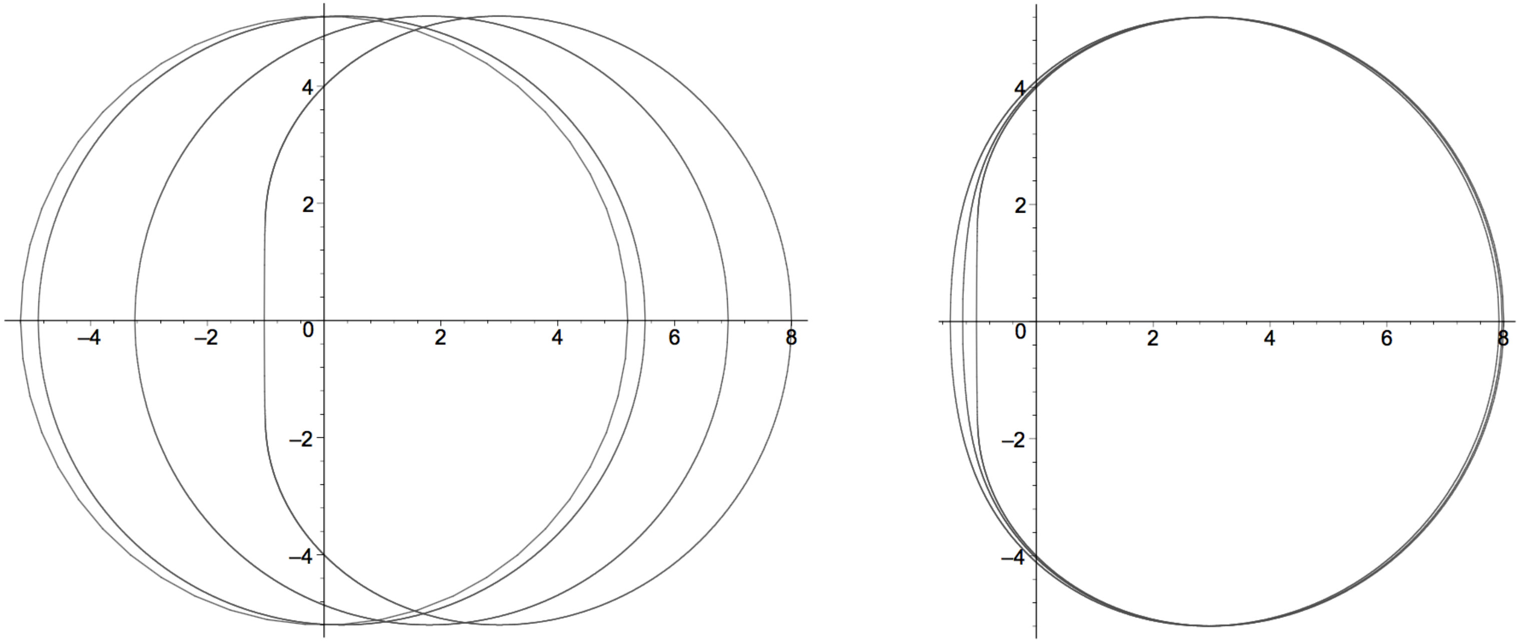
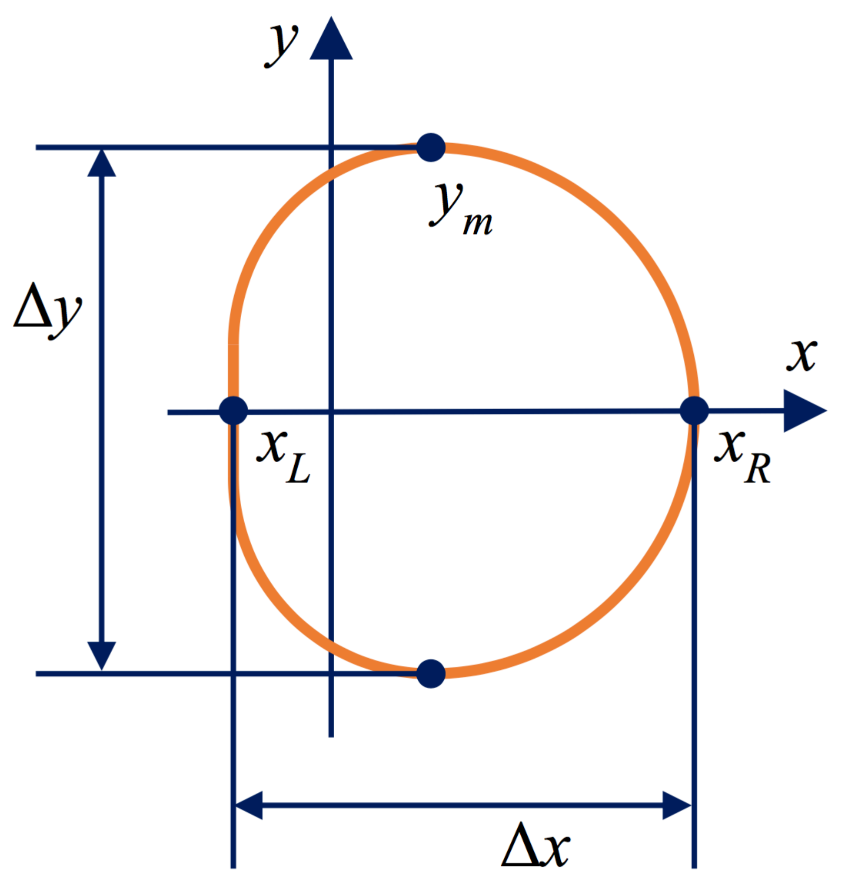


FIG. 3. LEFT: The shadow curves for the distant equatorial observer for (from the leftmost to the rightmost) $a = 0$, $0.1m$, $0.6m$, $0.9999m$. RIGHT: The shadow curves for $a = 0.97m$, $0.99m$, $0.9999m$. There is a notable difference in location of left borders, whereas the right borders are approximately at the same place, see (10) and (11).

If you can notice the non-sphericity with your own eyes, this means that you are looking at the shadow of nearly extreme black hole.

Extraction of BH spin using oblateness of the shadow



The shadow angular size gives us information about the **BH mass**.

The second thing which we could hope to measure is oblateness. The oblateness can give us information about the **BH spin**.

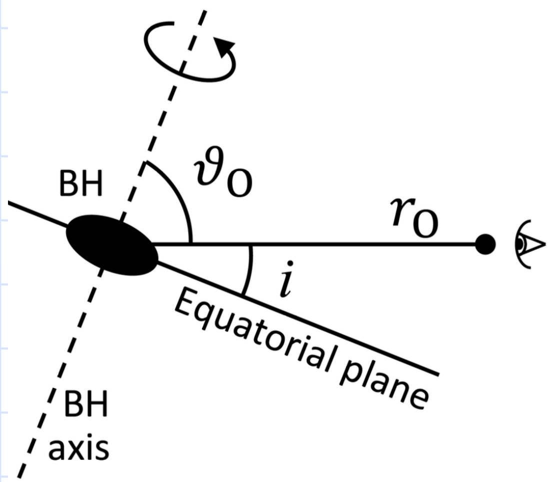
It is important also that the deformation depends on the viewing angle of observer: for the equatorial observer the deformation is strongest, while for the polar observer the deformation is absent.

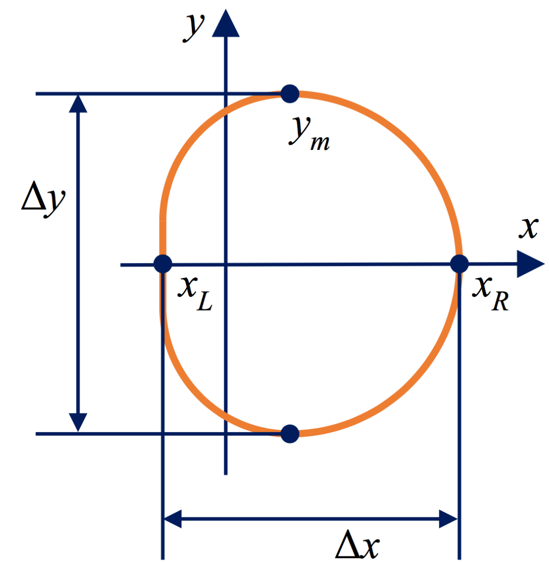
Shadow curve is written in terms of two variables which are functions of some parameter. Usually it is supposed that for extraction of spin we need to construct or model the entire curve of the shadow. We suggest fully analytical treatment which is easy to use.

Fully analytical calculation of spin via oblateness of the shadow

O.Yu. Tsupko, Physical Review D (2017)

The deformation is significant in case the black hole is nearly extreme and observer is not so far from the equatorial plane.
In this approximation, we present:





(i) the spin lower limit via oblateness,

$$a = (1-\delta)m\,, \quad \delta = 18\left(k - \frac{\sqrt{3}}{2}\right)^2, \quad k = \frac{\Delta x}{\Delta y}$$

(ii) the spin via oblateness and viewing angle, in case the latter is known from other observations.

$$\delta = 18\left(k - \frac{\sqrt{3}}{2}\right)^2 - 2k\left(k - \frac{\sqrt{3}}{2}\right) i^2$$

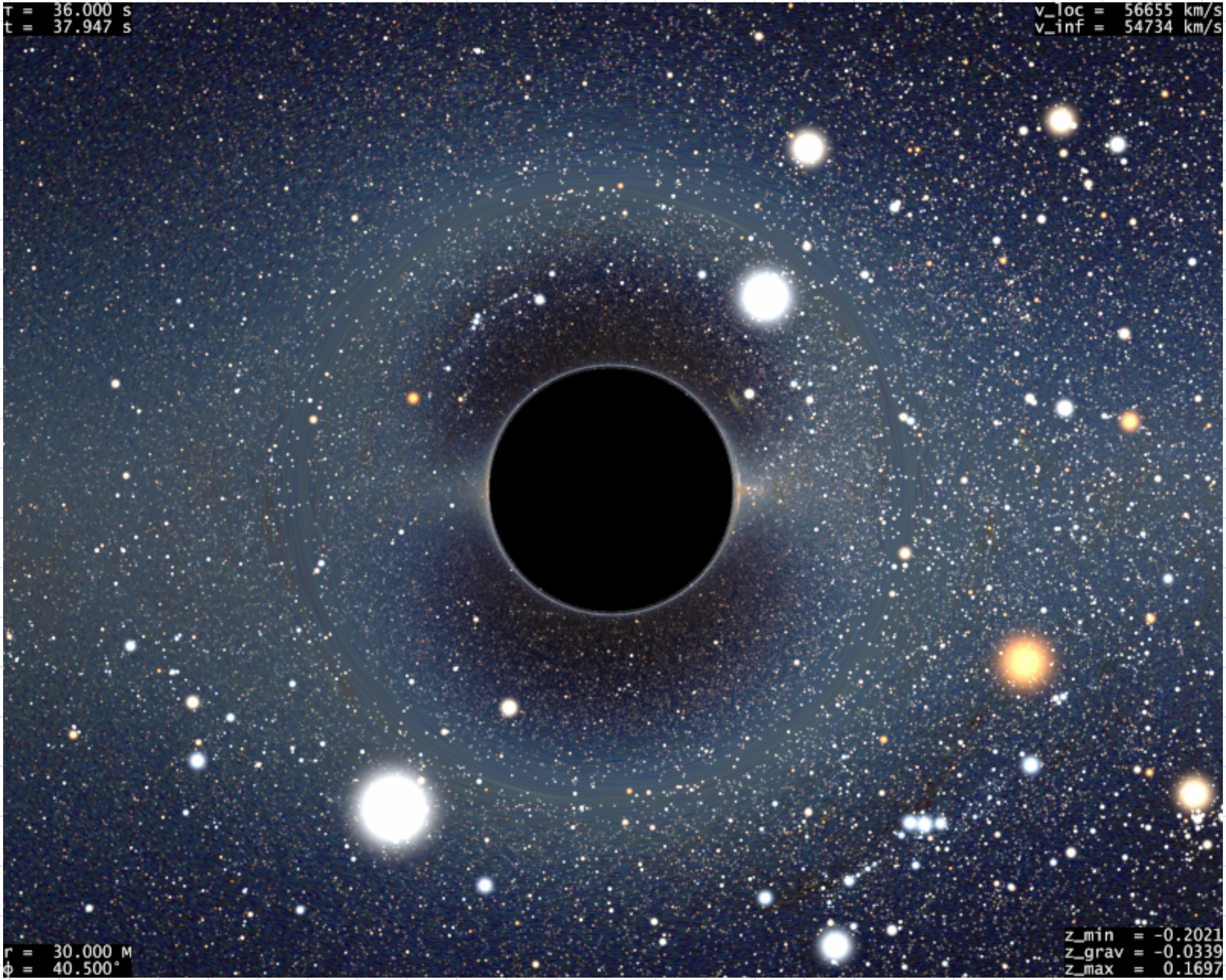
Influence of matter around BH on the observed size of the shadow

We would like to investigate the influence of matter around of BH on the observed size of the shadow.

Modeling is made by different groups.

For example, H. Falcke, F. Melia, and E. Agol, Astrophys. J. 528, L13 (2000).

Also, ray tracing programs have been written for producing realistic images of a black hole surrounded by an accretion disk, e.g., for the movie *Interstellar*.
The numerical techniques used for this movie are described in detail in O. James, E. Tunzelmann, P. Franklin, and K. Thorne, Classical Quantum Gravity 32, 065001 (2015).



Riazuelo A 2014 Simulation of starlight lensed by a camera orbiting a Schwarzschild black hole
(www2.iap.fr/users/riazuelo/interstellar)

Interstellar, courtesy of Warner Bros. Entertainment Inc..

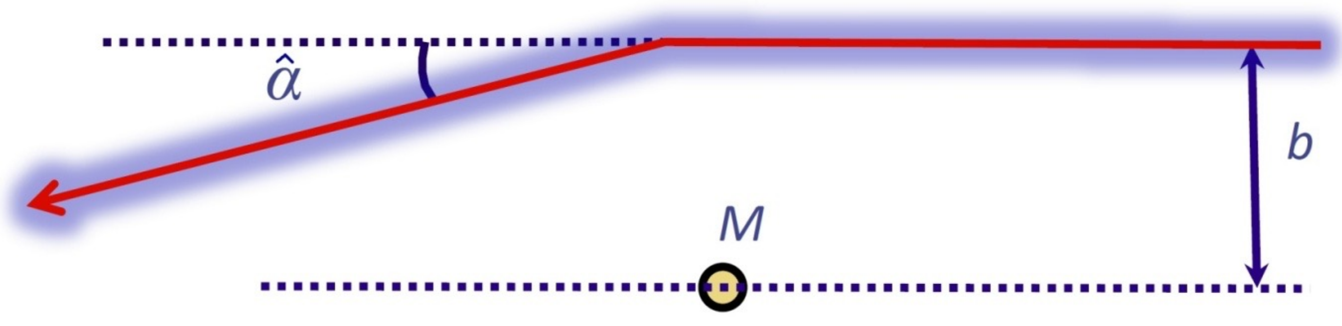
BH shadow in presence of plasma, analytical approach

We perform the first attempt of **analytical** investigation of plasma influence on the shadow size, in frame of **geometrical optics**, taking into account effects of general relativity and plasma presence.

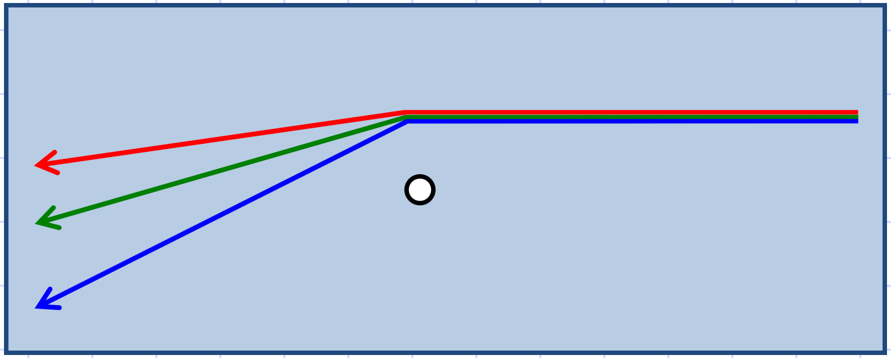
In this approximation, the presence of the plasma leads only to a **change of the geometrical size** of the shadow via a change of the light ray trajectories in this medium.

We consider general spherically symmetric space-time, and spherically symmetric distribution of plasma. Plasma is considered as a **medium with a given index of refraction**. Masses of plasma particles are not taken into account.

In vacuum the deflection angle of the photon does not depend on the photon frequency:



In presence of both plasma and gravity the deflection angle do depend on the photon frequency:



BH shadow in presence of plasma, analytical approach

We consider general spherically symmetric space-time,

$$g_{ik}dx^i dx^k = -A(r)dt^2 + B(r)dr^2 + D(r)(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2)$$

and spherically symmetric distribution of plasma:

$$n(r,\omega)^2 = 1 - \frac{\omega_p(r)^2}{\omega^2} \qquad \omega_p(r)^2 = \frac{4\pi e^2}{m} N(r)$$

Photon frequency depends on space coordinates in presence of gravity (gravitational redshift)

$$\omega = \omega(r)\,!$$

electron plasma frequency

number density of the electrons in the plasma

We have derived analytical formula for angular size of the shadow of BH surrounded by spherically symmetric plasma distribution and succeeded to rewrite it in compact way:

Angular radius $\alpha_{\rm sh}$ of shadow:

$$\boxed{\sin^2 \alpha_{\rm sh} = \frac{h(r_{\rm ph})^2}{h(r_{\rm O})^2}}$$

$r_{\rm ph}$ is the radius of photon sphere

$r_{\rm O}$ is the observer position

function $h(r)$ contains all information about metric, plasma distribution and photon frequency:

$$h(r)^2 = \frac{D(r)}{A(r)} \left(1 - A(r) \frac{\omega_p(r)^2}{\omega_0^2} \right)$$

$$g_{ik} dx^i dx^k = -A(r) dt^2 + B(r) dr^2 + D(r) (d\vartheta^2 + \sin^2 \vartheta \, d\varphi^2)$$

Condition for photon sphere:

$$\boxed{\frac{d}{dr} h(r)^2 \Big|_{r=r_{\rm ph}} = 0 \quad \Rightarrow \quad r_{\rm ph}}$$

Dependence on the photon frequency, significant for radio waves

V. Perlick, O.Yu. Tsupko, G.S. Bisnovatyi-Kogan, Phys. Rev. D 92, 104031 (2015)

Angular radius α_{sh} of shadow:

$$\boxed{\sin^2\alpha_{\mathrm{sh}} = \frac{h(r_{\mathrm{ph}})^2}{h(r_{\mathrm{O}})^2}}$$

$$h(r)^2 = \frac{D(r)}{A(r)}\left(1 - A(r)\frac{\omega_p(r)^2}{\omega_0^2}\right)$$

Using this formula, it is possible to calculate analytically an angular radius of the BH shadow:

for any spherically symmetric metric, for example Schwarzschild BH, without approximation of weak field

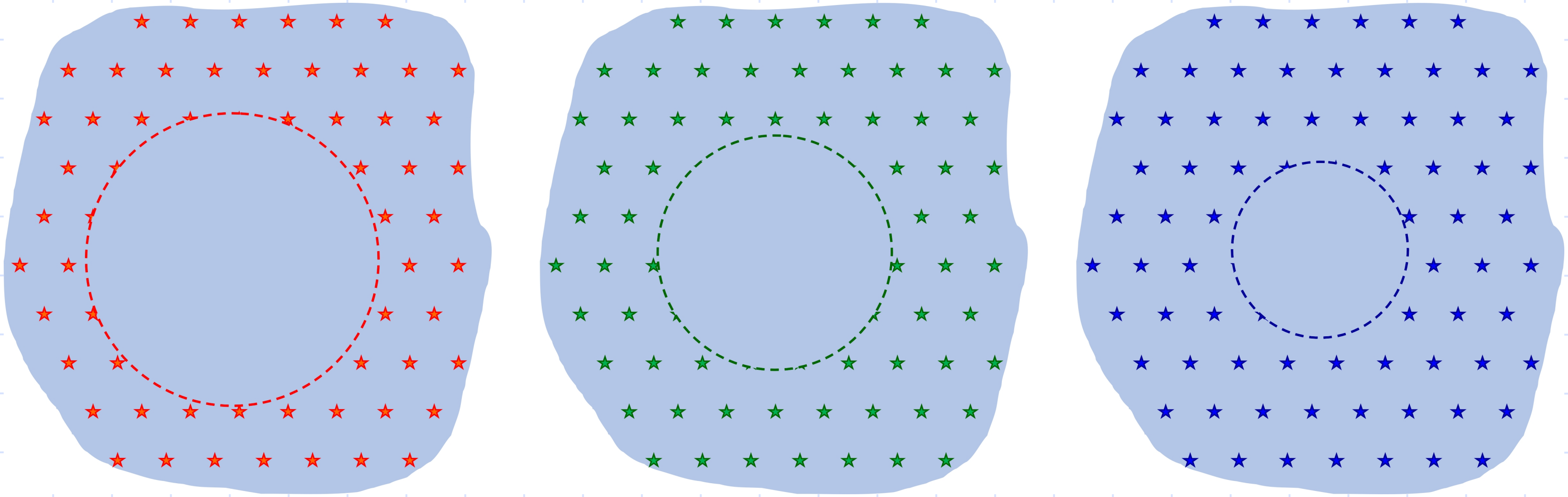
for any position of observer, in particular very close to BH and very far from BH

for any spherically symmetric distribution of plasma

for any photon frequency

Note that plasma is a dispersive medium therefore the radius of the shadow depends on the photon frequency (more rigorously, on the ratio of the plasma frequency and the photon frequency), effect is significant for very long radio waves

RAINBOW SHADOW!

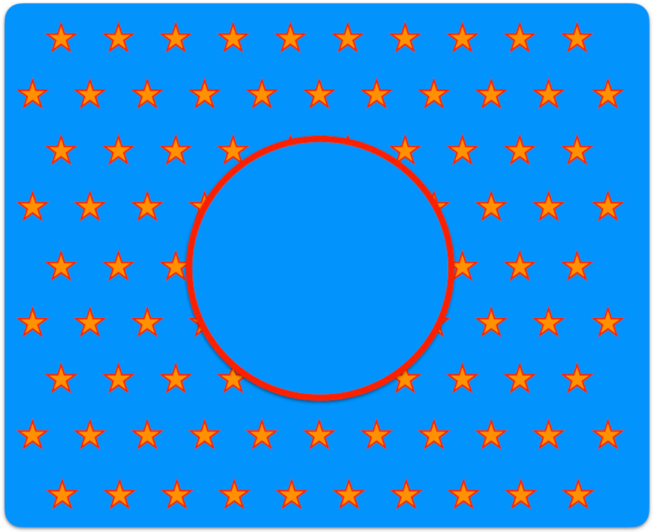


Effects of plasma on the shadow:

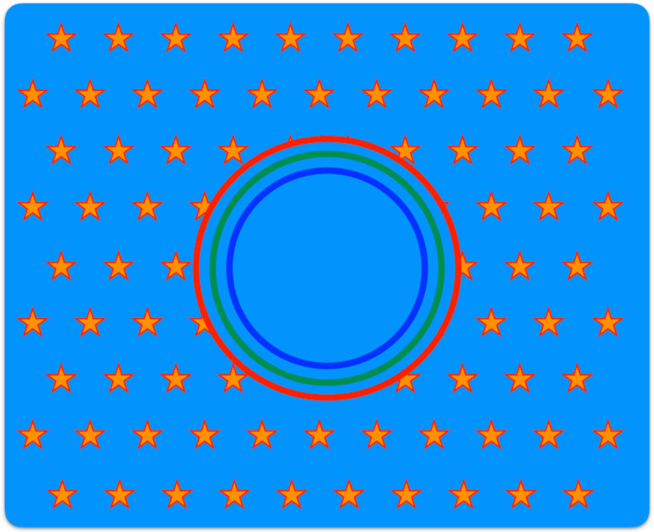
In the presence of a plasma the size of the shadow **depends on the wavelength** at which the observation is made, in contrast to the vacuum case where it is the same for all wavelengths.

The effect of the plasma is significant only in the **radio** regime.

For an observer far away from a Schwarzschild black hole the non-homogeneous plasma has a **decreasing effect** on the size of the shadow.



shadow in vacuum



shadow in plasma

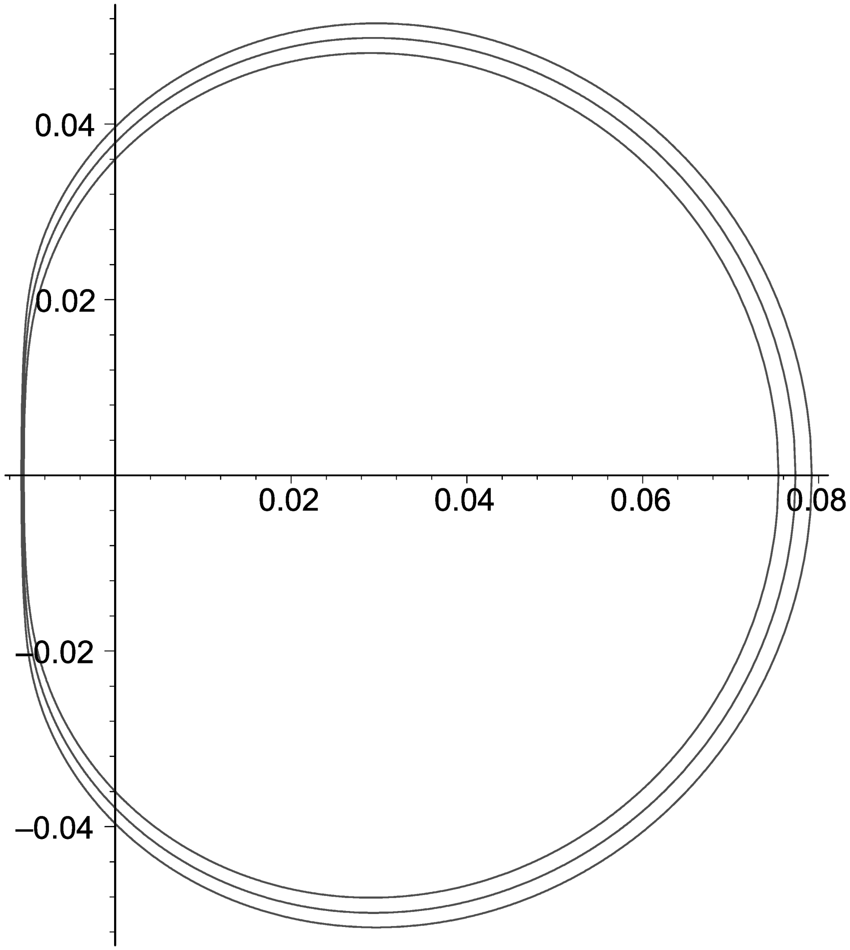
For Sgr A* and M87 effect is rather small.

Effect may be significant for radio waves but observations of the shadow are performed in the (sub)millimeter regime because at wavelengths of more than about 1.3 mm it is expected that the shadow is washed out by scattering.

Influence of plasma on shadow of Kerr BH

V. Perlick and O.Yu. Tsupko, Physical Review D 95, 104003 (2017)

The same effects:
shadow becomes chromatic,
becomes smaller in case of
non-homogeneous plasma



We have demonstrated that the Hamilton-Jacobi equation is separable, i.e., that a generalized Carter constant exists, only for special distributions of the plasma electron density. The necessary and sufficient condition for separability is:

the plasma frequency is of the form

$$\omega_p(r,\vartheta)^2 = \frac{f_r(r) + f_\vartheta(\vartheta)}{r^2 + a^2 \cos^2\vartheta}$$

with some functions $f_r(r)$ and $f_\vartheta(\vartheta)$. Then

Conclusions

Angular size of the shadow —> the **BH mass.**

Oblateness of the shadow (the ratio of the horizontal and vertical angular diameters) —> the **BH spin**

Angular size in different wavelengths —> **plasma** environment around BH

Publications:

about the shadow:

1. V. Perlick, O.Yu. Tsupko and G.S. Bisnovatyi-Kogan, **Physical Review D** 92, 104031 (2015)

2. V. Perlick and O.Yu. Tsupko, **Physical Review D** 95, 104003 (2017)

3. O.Yu. Tsupko, **Physical Review D** 95, 104058 (2017)

other chromatic effects of gravitational lensing in plasma:

4. G.S. Bisnovatyi-Kogan, O.Yu. Tsupko, **Gravitation and Cosmology**, 15(1), 20-27 (2009).

5. G.S. Bisnovatyi-Kogan and O.Yu. Tsupko, **MNRAS** 404, 1790 (2010)

6. O.Yu. Tsupko and G.S. Bisnovatyi-Kogan, **Physical Review D** 87, 124009 (2013)

7. O. Yu. Tsupko, **Physical Review D** 89, 084075 (2014)

8. G. S. Bisnovatyi-Kogan and O. Yu. Tsupko, **Plasma Physics Reports**, 2015, Vol. 41, No. 7, pp. 562–581 (review, arXiv:1507.08545)