Influence of a plasma on the black hole shadow

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**Einstein angle:**

\[
\hat{\alpha} = \frac{4GM}{c^2b} = \frac{2R_S}{b}
\]

\(b \gg R_S = \frac{2GM}{c^2}\)

Deflection angle of the photon in vacuum does not depend on the photon frequency (or energy). Deflection in vacuum is **achromatic**.

Gravitational lensing in vacuum is achromatic.
Gravitational lensing in vacuum, the simplest scheme

At this picture there is the example of the simplest model of Schwarzschild point-mass lens. Observer sees two point images of source instead of one single real point source.
How is this situation changed in presence of plasma?

In outer space, the rays of light travel through the plasma. In plasma, photons undergo various effects. For gravitational lensing the main interest is the change in the angle of deflection of the light ray. Due to dispersion properties of plasma, we may expect that chromatic effects will arise.
Light rays in a transparent, inhomogeneous medium propagate along curved trajectories. This phenomenon is called refraction and is well known from everyday life.

The bending of the light rays due to refraction is not related to relativity or gravity and occurs only if the medium is optically non-homogeneous.

\[ n = n(x) \]

‘Non-homogeneous’ means here that the refractive index depends explicitly on space coordinates.
Simplified consideration of the problem: 'linearized' combination of gravity and plasma

The simplest way to add plasma to the problem of gravitational lensing is to assume both deflection angles due to gravity and due to refraction in the plasma small.

**Total deflection is:**

- vacuum gravitational deflection
  
  with using of the linearized theory of gravitation (approximate Einstein formula)
  
  +

- refractive deflection in non-homogeneous plasma
  
  (assuming that the refractive index differs only slightly from unity (vacuum))

In this approximation, the joint action of gravitation and refraction was considered from the 1960s, with reference to the propagation of radio signals in the solar corona.
Why we are interested in more rigorous approach

1) **Black holes** and other compact objects **surrounded** by a plasma.

Deflection angles are not small, the linearized theory of gravity is insufficient.

2) The interesting question is whether the **gravitational deflection itself** changes in the presence of a **medium** around a gravitating body instead of a vacuum.

Also, there are many more complex lens models (not just a point mass).
Geometrical optics in presence both gravity and plasma

We need general theory for geometrical optics in arbitrary medium (dispersive or not) in curved space-time (in presence of gravity).

The first self-consistent approach for geometrical optics in medium in presence of gravity. See also:
J. Bicˇa´k and P. Hadrava (1975)

The first monograph completely devoted to review of general-relativistic ray optics in medium:

Propagation of electromagnetic waves in magnetised plasma:

On language of gravitational lensing the problem of plasma influence is considered for the first time:
is the gravitational deflection of the light rays in the medium the same as in vacuum?

We mean here the \textit{homogeneous} medium, so there is no refraction.

\textbf{Answer is:}
deflection is \textbf{the same} as in vacuum, if medium is \textbf{non-dispersive} \((n=\text{const})\)
And it \textbf{differs}, if medium is \textbf{dispersive}! \((n=n(\omega))\)

The physical reason is a dependence of the wave frequency on space coordinates in presence of gravity (gravitational redshift)

Bisnovatyi-Kogan and Tsupko (2009, 2010)
We have shown for the first time, that the gravitational deflection in homogeneous plasma differs from the vacuum deflection angle, and depends on frequency of the photon:

\[ n^2 = 1 - \frac{\omega_e^2}{[\omega(r)]^2}, \quad \omega_e^2 = \frac{4\pi e^2 N_0}{m} = \text{const} \]

Here, the frequency of the photon \( \omega(r) \) depends on the spatial coordinate \( r \) due to the presence of a gravitational field (the gravitational redshift). We will use the following notation: \( \omega(\infty) \equiv \omega \), \( e \) is the charge of the electron, \( m \) is the electron mass, \( \omega_e \) is the electron plasma frequency, and \( N_0 = \text{const} \) is the electron concentration in a homogeneous plasma. This for-
Instead of two concentrated images with complicated spectra, we will have two ‘rainbow’ images, formed by the photons with different frequencies, which are deflected by different angles.

Point-mass gravitational lens in homogeneous plasma: it acts like spectrometer!

Effect is significant only for radiowaves

Different colors mean different wavelengths
Gravitational lensing in non-homogeneous plasma:

In non-homogeneous plasma two effects should be taken into account:

1) Difference of gravitational deflection from vacuum case due to plasma presence (which we have just discussed)

2) Refraction (usually bigger)

Both effects are chromatic in plasma
Total deflection angle (in weak deflection approximation)

Additional correction to the gravitational deflection due to plasma presence. It depends on the photon frequency. It takes place both in homogeneous and inhomogeneous plasma.

The refraction connected with the plasma inhomogeneity. It depends on the photon frequency because the plasma is dispersive medium. This angle equals to zero if the plasma is homogeneous.

\[
\hat{\alpha} = \alpha_{einst} + \alpha_{add} + \alpha_{refr}
\]

\[
\frac{2R_s}{b} \quad \propto \frac{R_s}{b} \frac{\omega_e^2}{\omega^2} \quad \propto \nabla \frac{\omega_e^2}{\omega^2}
\]

Vacuum gravitational deflection (Einstein)

Bisnovatyi-Kogan and Tsupko (2009, 2010)
Observational predictions:

- Xinzhong Er, Shude Mao, 2014: angular difference between optical and radio images is up to $10^{-2}$ arcsec due to presence of inhomogeneous plasma for 375 MHz
What we propose for observations:

1) Compare observations of strong lens system with multiple images in optical and radio band, or compare observations in two radio bands

2) Shift of angular position of every image can be observed

3) As a result: investigation of plasma properties in vicinity of lens
Shadow of supermassive black holes

SMBH in the center of galaxies, for example, in the center of our galaxy

A distant observer should “see” this black hole as a dark disk in the sky which is known as the “shadow”

For the black hole at the center of our galaxy, size of the shadow is about 53 $\mu$as (size of grapefruit on the Moon).

At present, two projects are under way to observe this shadow which would give important information on the compact object at the center of our galaxy. These projects, which are going to use (sub)millimeter VLBI observations with radio telescopes distributed over the Earth, are the Event Horizon Telescope (http://eventhorizontelescope.org) and the BlackHoleCam (http://blackholecam.org).
What is a shadow of a black hole?

We consider a black hole as a region of very strong curved space-time, and the boundary of this region is the event horizon.

Nothing surprising that we expect to see some black ‘spot’ in the sky where we suggest a black hole is situated.

But why people say that ‘we should see a shadow’ instead of ‘we should see a black hole’?

Answer is that due to STRONG bending of light rays coming to us, we see something very different from ‘real view’ of black hole.

In case of spherically symmetric black hole a difference is only in angular size, in Kerr metric (which is axially symmetric) picture become non-symmetrical.
On the theoretical side, the shadow is defined as the region of the observer’s sky that is left dark if there are light sources distributed everywhere but not between the observer and the black hole.
Angular size of the shadow of Schwarzschild BH in vacuum

\[ \sin^2 \alpha_{\text{sh}} = \frac{27M^2(1 - 2M/r_O)}{r_O^2} \]

For distant observer, \( r_O \gg M \):

\[ \alpha_{\text{sh}} \approx 3\sqrt{3} \frac{M}{r_O} \]
Shadow is not the Euclidian image of the region inside the event horizon

Shadow is not the Euclidian image of the region inside the photon sphere

Shadow is the image of the region inside the photon sphere increased by bending of light rays
For **rotating** black hole the shadow is not symmetrical although metric is axially symmetric. Shadow is **deformed** and **oblate**.
Analytical calculation of the shadow size and shape in vacuum, other metrics:

**Kerr:**


(for observer far away from a black hole)

**Class of Plebański-Demiański spacetimes:**


(including Kerr space-time and arbitrary position of observer)
If you can notice the non-sphericity with your own eyes, this means that you are looking at the shadow of nearly extreme black hole.
The shadow angular size gives us information about the BH mass.

The second thing which we could hope to measure is oblateness. The oblateness can give us information about the BH spin.

It is important also that the deformation depends on the viewing angle of observer: for the equatorial observer the deformation is strongest, while for the polar observer the deformation is absent.

Shadow curve is written in terms of two variables which are functions of some parameter. Usually it is supposed that for extraction of spin we need to construct or model the entire curve of the shadow. We suggest fully analytical treatment which is easy to use.
The deformation is significant in case the black hole is nearly extreme and observer is not so far from the equatorial plane. In this approximation, we present:

(i) the spin lower limit via oblateness,

\[ a = (1 - \delta)m, \quad \delta = 18 \left( k - \frac{\sqrt{3}}{2} \right)^2, \quad k = \frac{\Delta x}{\Delta y} \]

(ii) the spin via oblateness and viewing angle, in case the latter is known from other observations.

\[ \delta = 18 \left( k - \frac{\sqrt{3}}{2} \right)^2 - 2k \left( k - \frac{\sqrt{3}}{2} \right) i^2 \]
We would like to investigate the influence of matter around of BH on the observed size of the shadow.

Modeling is made by different groups.


Also, ray tracing programs have been written for producing realistic images of a black hole surrounded by an accretion disk, e.g., for the movie *Interstellar*.

The numerical techniques used for this movie are described in detail in O. James, E. Tunzelmann, P. Franklin, and K. Thorne, Classical Quantum Gravity 32, 065001 (2015).
Riazuelo A 2014 Simulation of starlight lensed by a camera orbiting a Schwarzschild black hole
(www2.iap.fr/users/riazuelo/interstellar)
We perform the first attempt of *analytical* investigation of plasma influence on the shadow size, in frame of *geometrical optics*, taking into account effects of general relativity and plasma presence.

In this approximation, the presence of the plasma leads only to a change of the geometrical size of the shadow via a change of the light ray trajectories in this medium.

We consider general spherically symmetric space-time, and spherically symmetric distribution of plasma. Plasma is considered as a medium with a given index of refraction. Masses of plasma particles are not taken into account.
In vacuum the deflection angle of the photon does not depend on the photon frequency:

\[ \hat{\alpha} \]

In presence of both plasma and gravity the deflection angle do depend on the photon frequency:

\[ b \]

\[ M \]
BH shadow in presence of plasma, analytical approach

We consider general spherically symmetric space-time,

\[ g_{ik}dx^i dx^k = -A(r)dt^2 + B(r)dr^2 + D(r)(d\theta^2 + \sin^2\theta d\varphi^2) \]

and spherically symmetric distribution of plasma:

\[ n(r, \omega)^2 = 1 - \frac{\omega_p(r)^2}{\omega^2} \quad \omega_p(r)^2 = \frac{4\pi e^2}{m} N(r) \]

Photon frequency depends on space coordinates in presence of gravity (gravitational redshift)

\[ \omega = \omega(r) ! \]

electron plasma frequency

number density of the electrons in the plasma
We have derived analytical formula for angular size of the shadow of BH surrounded by spherically symmetric plasma distribution and succeeded to rewrite it in compact way:

**Angular radius \( \alpha_{sh} \) of shadow:**

\[
\sin^2 \alpha_{sh} = \frac{h(r_{ph})^2}{h(r_O)^2}
\]

\( r_{ph} \) is the radius of photon sphere

\( r_O \) is the observer position

Function \( h(r) \) contains all information about metric, plasma distribution and photon frequency:

\[
h(r)^2 = \frac{D(r)}{A(r)} \left( 1 - A(r) \frac{\omega_p(r)^2}{\omega_0^2} \right)
\]

\( g_{ik} dx^i dx^k = -A(r) dt^2 + B(r) dr^2 + D(r) (d\theta^2 + \sin^2 \theta d\phi^2) \)

**Condition for photon sphere:**

\[
\frac{d}{dr} h(r)^2 \bigg|_{r=r_{ph}} = 0
\]

\( r_{ph} \)

 Dependence on the photon frequency, significant for radio waves
Angular radius $\alpha_{\text{sh}}$ of shadow:

$$\sin^2 \alpha_{\text{sh}} = \frac{h(r_{\text{ph}})^2}{h(r_O)^2}$$

$$h(r)^2 = \frac{D(r)}{A(r)} \left(1 - A(r) \frac{\omega_p(r)^2}{\omega_0^2}\right)$$

Using this formula, it is possible to calculate analytically an angular radius of the BH shadow:

- for any spherically symmetric metric, for example Schwarzschild BH, without approximation of weak field
- for any position of observer, in particular very close to BH and very far from BH
- for any spherically symmetric distribution of plasma
- for any photon frequency
Note that plasma is a dispersive medium therefore the radius of the shadow depends on the photon frequency (more rigorously, on the ratio of the plasma frequency and the photon frequency), effect is significant for very long radio waves.
Effects of plasma on the shadow:

In the presence of a plasma the size of the shadow depends on the wavelength at which the observation is made, in contrast to the vacuum case where it is the same for all wavelengths.

The effect of the plasma is significant only in the radio regime.

For an observer far away from a Schwarzschild black hole the non-homogeneous plasma has a decreasing effect on the size of the shadow.
For Sgr A* and M87 effect is rather small.

Effect may be significant for radio waves but observations of the shadow are performed in the (sub)millimeter regime because at wavelengths of more than about 1.3 mm it is expected that the shadow is washed out by scattering.
Influence of plasma on shadow of Kerr BH

The same effects:
shadow becomes chromatic,
becomes smaller in case of
non-homogeneous plasma

We have demonstrated that the Hamilton-Jacobi equation is separable, i.e., that a generalized Carter constant exists, only for special distributions of the plasma electron density. The necessary and sufficient condition for separability is:

the plasma frequency is of the form

\[ \omega_p(r, \vartheta)^2 = \frac{f_r(r) + f_{\vartheta}(\vartheta)}{r^2 + a^2 \cos^2 \vartheta} \]

with some functions \( f_r(r) \) and \( f_{\vartheta}(\vartheta) \). Then
Conclusions

Angular size of the shadow —> the BH mass.

Oblateness of the shadow (the ratio of the horizontal and vertical angular diameters) —> the BH spin

Angular size in different wavelengths —> plasma environment around BH
Publications:

about the shadow:


other chromatic effects of gravitational lensing in plasma:


